FREE EDITION · NOTES + 3 SAMPLE MCQS

CUET · MATHEMATICS · CLASS XI · CODE 319

Binomial Theorem

CUET unit: Binomial Theorem

By **UniDrill** · NCERT-grounded study material

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📌 Snapshot

- Establishes the Binomial Theorem for positive integral indices: a compact rule to expand `(a + b)^n` without repeated multiplication, motivated by the difficulty of evaluating numbers like `(98)^5` or `(101)^6` by hand (NCERT §7.1, p. 126).
- Develops the pattern empirically — observations about number of terms, decreasing/increasing powers, and constant sum of indices — then reorganises the coefficients into Pascal's triangle / Meru Prastara (NCERT §7.2, pp. 126–128).
- Recasts Pascal's triangle entries as combinations `nCr` and proves the theorem `(a+b)^n = Σ nCk a^(n-k) b^k` by mathematical induction (NCERT §7.2.1, p. 129).
- Derives the standard special-case expansions `(x − y)^n`, `(1 + x)^n`, `(1 − x)^n`, and uses them to obtain identities such as `2^n = nC0 + nC1 + … + nCn` and `0 = nC0 − nC1 + … + (−1)^n nCn` (NCERT §7.2.2, pp. 130–131).
- CUET tests direct expansion, specific-term identification using `nCk a^(n-k) b^k`, numerical approximation problems like `(98)^5` and `(0.99)^5`, and divisibility/identity proofs.

📖 Detailed Notes

2.1 Core concepts

- The Binomial Theorem here applies to positive integral indices only; the general case for integral or rational `n` is out of scope (NCERT §7.1, p. 126). General binomial series for non-integer exponents involve infinite series and convergence considerations.
- Motivation: numerical computation of high powers such as `(98)^5` or `(101)^6` by repeated multiplication is laborious, so a single formulaic expansion is sought (NCERT §7.1, p. 126).
- The expansions of `(a + b)^0`, `(a + b)^1`, `(a + b)^2`, `(a + b)^3`, `(a + b)^4` are tabulated and three structural observations are extracted: (i) number of terms is one more than the index, (ii) the power of `a` decreases by 1 and the power of `b` increases by 1 in successive terms, (iii) the sum of indices of `a` and `b` in every term equals the index of `(a + b)` (NCERT §7.2, p. 126).

- The coefficients are arranged in a triangular array — 1 at the apex, with each interior entry obtained by adding the two entries immediately above it — known as Pascal's triangle, also called Meru Prastara after Pingla (NCERT §7.2, pp. 127–128). The triangle predates Pascal; Indian mathematician Pingla described it in his Chandahsastra (~200 BCE).
- For ready use, Pascal's triangle is rewritten with entries as combinations `nCr = n! / [r!(n − r)!]`, `0 ≤ r ≤ n`, with `nC0 = 1 = nCn`; this lets us write the row for any index without writing all preceding rows (NCERT §7.2, p. 128).
- For index 7, the row is `7C0, 7C1, 7C2, 7C3, 7C4, 7C5, 7C6, 7C7`, giving `(a + b)^7 = 7C0 a^7 + 7C1 a^6 b + 7C2 a^5 b^2 + 7C3 a^4 b^3 + 7C4 a^3 b^4 + 7C5 a^2 b^5 + 7C6 a b^6 + 7C7 b^7` (NCERT §7.2, p. 128).
- **Binomial Theorem statement (§7.2.1):** For any positive integer `n`, `(a + b)^n = nC0 a^n + nC1 a^(n-1) b + nC2 a^(n-2) b^2 + … + nC(n-1) a b^(n-1) + nCn b^n`, equivalently `(a + b)^n = Σ_{k=0}^{n} nCk a^(n-k) b^k` (NCERT §7.2.1, pp. 129–130).
- **Proof** is by the principle of mathematical induction: base step at `n = 1`; inductive step uses `kCr + kC(r-1) = (k+1)Cr` together with `kC0 = (k+1)C0 = 1` and `kCk = (k+1)C(k+1) = 1` to pass from `P(k)` to `P(k+1)` (NCERT §7.2.1, p. 129). Pascal's identity is the algebraic engine of the inductive step.
- Worked illustration: `(x + 2)^6 = x^6 + 12x^5 + 60x^4 + 160x^3 + 240x^2 + 192x + 64` (NCERT §7.2.1, p. 129). Note that powers of 2 grow as 1, 2, 4, 8, 16, 32, 64 while binomial coefficients are 1, 6, 15, 20, 15, 6, 1.
- **Five Observations on the expansion (§7.2.1):** sigma form, the term `nCr` is called a binomial coefficient, there are `(n + 1)` terms, the index of `a` runs `n, n-1, …, 0` while the index of `b` runs `0, 1, …, n`, and the sum of the indices of `a` and `b` in every term equals `n` (NCERT §7.2.1, p. 130).
- **Special case 1 — `(x − y)^n`:** Setting `a = x, b = -y` gives `(x − y)^n = nC0 x^n − nC1 x^(n-1) y + nC2 x^(n-2) y^2 − … + (-1)^n nCn y^n`; the signs alternate. Illustration: `(x − 2y)^5 = x^5 − 10x^4 y + 40x^3 y^2 − 80x^2 y^3 + 80 x y^4 − 32 y^5` (NCERT §7.2.2, p. 130).
- **Special case 2 — `(1 + x)^n`:** Setting `a = 1, b = x` gives `(1 + x)^n = nC0 + nC1 x + nC2 x^2 + … + nCn x^n`. In particular, putting `x = 1` yields `2^n = nC0 + nC1 + nC2 + … + nCn` (NCERT §7.2.2, pp. 130–131).
- **Special case 3 — `(1 − x)^n`:** Setting `a = 1, b = −x` gives `(1 − x)^n = nC0 − nC1 x + nC2 x^2 − … + (-1)^n nCn x^n`. Putting `x = 1` yields `0 = nC0 − nC1 + nC2 − … + (-1)^n nCn` (NCERT §7.2.2, p. 131).
- **Numerical applications (§7.2.2 examples):** `(98)^5` is computed as `(100 − 2)^5 = 9 039 207 968`; `(1.01)^1000000 > 10 000` by retaining the first two non-negative binomial terms `1 + 1 000 000 × 0.01` (NCERT pp. 131–132).
- **Divisibility application (Example 4):** Using `(1 + 5)^n = 1 + 5n + 5^2·nC2 + 5^3·nC3 + … + 5^n`, one obtains `6^n − 5n = 25k + 1` where `k = nC2 + 5·nC3 + … +`

$5^{(n-2)}$, proving that $6^n - 5n$ leaves remainder 1 on division by 25 (NCERT §7.2.2, p. 132).

- The triangular array of binomial coefficients is called Pascal's triangle (NCERT Summary, p. 133).
- The general term of $(a + b)^n$ — the (r + 1)-th term — is $T_{r+1} = nCr\ a^{(n-r)} b^r$. This single formula handles every "find coefficient of x^k" and "find middle term" problem.
- Two identities from Permutations and Combinations (Ch. 6) are useful here: symmetry nCr = nC(n−r) and Pascal's identity nCr + nC(r−1) = (n+1)Cr. These let you compute large nCr by reducing to small r quickly.
- "Middle term" considerations: for the binomial $(a + b)^n$, if n is even there is one middle term at position (n/2) + 1; if n is odd there are two middle terms at positions (n+1)/2 and (n+3)/2.
- The ratio of consecutive terms is $T_{r+2}/T_{r+1} = [(n - r)/(r + 1)] \cdot (b/a)$. This ratio is monotonic and helps locate the largest term (greatest binomial coefficient times power).
- The greatest binomial coefficient: in row n of Pascal's triangle, the largest entry is $nC_{\lfloor n/2 \rfloor}$ (one or two values for even/odd n). For example, the largest binomial coefficient in $(a + b)^{10}$ is 10C5 = 252.
- Connection to combinatorics: nCk counts the number of k-element subsets of an n-element set; the binomial theorem can be read as a generating-function identity that encodes all of these subset counts simultaneously.
- The numerical approximation philosophy: for $(1 + x)^n$ with |x| small, the first two terms 1 + nx are an excellent approximation; this is the leading-order Taylor approximation, formally justified in Class XII analysis but already useful here.
- A useful identity derived from sub-cases of the binomial theorem: $\Sigma\ k \cdot nCk = n \cdot 2^{(n-1)}$ (obtained by differentiating $(1+x)^n$ and setting x = 1). This and similar identities show how calculus tools enrich combinatorial results.
- Connection to probability: the binomial probability distribution $P(X = k) = nCk\ p^k (1-p)^{(n-k)}$ has its mass function built directly from the binomial theorem with a = p, b = 1−p, giving $\Sigma_k P(X = k) = (p + 1 - p)^n = 1$.
- For a quick "row of Pascal's triangle" without computing all factorials: start with 1 at both ends; compute each interior entry as the sum of the two above. This builds up row by row efficiently for small n ($\leqslant$ 15).
- The theorem extends conceptually but not computationally to $(a + b + c)^n$ via the multinomial theorem; this is beyond Class XI but appears occasionally in JEE-style questions.

2.2 Definitions to memorise

Term	Definition	Page
Binomial Theorem	(a+b)^n = ΣnCk a^(n-k) b^k	129
Sigma form	Σ_{k=0}^{n} nCk a^(n-k) b^k	130
Binomial coefficient	nCr = n!/(r!(n-r)!)	130
Number of terms	n + 1	130
Sum-of-indices	Always equals n	130
Pascal's triangle	Triangular array of binomial coefficients	127
Meru Prastara	Pingla's name for the same triangle	128
General (r+1)-th term	T_{r+1} = nCr a^(n-r) b^r	130
(x − y)^n expansion	Alternating sign: nCk x^(n-k) (−y)^k	130
(1 + x)^n expansion	ΣnCk x^k	130
(1 − x)^n expansion	Σ(−1)^k nCk x^k	131
Sum of coefficients	2^n	131
Alternating sum	0	131
Pascal's identity	nCr + nC(r-1) = (n+1)Cr	129
Symmetry	nCr = nC(n−r)	117
Middle term (n even)	(n/2 + 1)-th term	130
Middle terms (n odd)	((n+1)/2)-th and ((n+3)/2)-th	130
Coefficient of x^k in (1+x)^n	nCk	131
Sum of even-indexed binomials	2^(n−1)	131
Sum of odd-indexed binomials	2^(n−1)	131
Numerical approximation use	E.g. (98)^5 = (100 − 2)^5	131
Divisibility use	E.g. 6^n − 5n ≡ 1 (mod 25)	132
Proof technique	Mathematical induction	129
nC0	1	128
nCn	1	128

2.3 Diagrams / processes to remember

- **Fig 7.1 (p. 127):** Tabular display of coefficients for indices 0 through 4 — the raw form from which Pascal's pattern is read off.
- **Fig 7.2 (p. 127):** Pascal's triangle with explicit arrows/triangular markers showing how adjacent entries in row `n` add to give the next entry in row `n+1`.

- **Fig 7.3 (p. 128):** Pascal's triangle rewritten with combination notation `kCr` (each cell shows `nCr` and the numerical value), exhibited up to row `n = 5`.
- **Portrait of Blaise Pascal (1623–1662)** on p. 126 alongside the introduction, linking the topic to its historical figure.
- **Process for expanding using Pascal's triangle:** pick the row matching the index, attach decreasing powers of the first quantity and increasing powers of the second (worked example on p. 127 for `(2x + 3y)^5`).
- **Process — find (r+1)-th term:** identify n and the binomial (a+b)^n; substitute into T_{r+1} = nCr a^(n-r) b^r; simplify.
- **Process — find coefficient of x^k in (a + bx)^n:** set up T_{r+1}, identify which r makes the x-exponent equal to k, solve for r, plug back.
- **Process — middle term:** if n even, middle term is the (n/2 + 1)-th; if n odd, there are two middle terms at positions (n+1)/2 and (n+3)/2.
- **Process — numerical approximation:** write the number as (a + b)^n where b is small; truncate the expansion after a few terms.

2.4 Common confusions / NTA trap points

- **Number of terms vs. index.** The expansion of `(a + b)^n` has `n + 1` terms, not `n`. The first-term index of `a` is `n`, not `n - 1` (NCERT §7.2.1 obs. 3–4, p. 130).
- **Sign pattern in `(x - y)^n` and `(1 - x)^n`.** Coefficients of even powers of the negative quantity stay positive, odd powers turn negative; students often misplace a sign. The `r`-th coefficient carries `(-1)^r`, not `(-1)^(n-r)` (NCERT §7.2.2, pp. 130–131).
- **Reading off the right term.** In `(a + b)^n = Σ nCk a^(n-k) b^k`, the term with `b^k` is the `(k + 1)`-th term, not the `k`-th — because the indexing starts at `k = 0`. A frequent NTA distractor is to swap `nCk a^(n-k) b^k` with `nCk a^k b^(n-k)`.
- **Sum-of-indices invariance.** Every term in the expansion has indices summing to `n`. An option in which the indices sum to `n - 1` or `n + 1` is automatically a distractor (NCERT §7.2.1 obs. 5, p. 130).
- **Pascal's triangle row vs. index.** Row labelled by index `n` contains `n + 1` entries; the first entry is `nC0 = 1`, not `nC1`. Misreading the row offset produces wrong coefficients (NCERT Fig 7.3, p. 128).
- **`(1.01)^1000000` style comparison.** Keep only the first one or two terms of `(1 + 0.01)^1000000` and discard the rest because they are positive — discarding a negative term would be illegitimate (NCERT Example 3, p. 132).
- **Middle term count.** For n odd, there are two middle terms, not one. CUET sometimes traps with a one-term option when n is odd.
- **Wrong substitution for x in (1 + x)^n.** To get the sum 2^n, substitute x = 1; substituting x = 2 gives 3^n, not 2^n.

- **Treating coefficient and term as the same.** "Coefficient of x^k" is the number multiplying x^k; the "term containing x^k" includes the x^k.
- **Forgetting the factor (b)^k in coefficient hunts.** When the binomial is (a + bx)^n with b ≠ 1, the coefficient of x^k is nCk·a^(n-k)·b^k.
- **Mis-applying the theorem to non-integer n.** The Class XI version is only for positive integer n; negative or fractional n is the general binomial series (not in syllabus).
- **Confusing nCr with nPr.** Combinations have an r! in the denominator; permutations do not.

2.5 Key formulas & theorems

Formula	Statement	NCERT page
Binomial Theorem	(a+b)^n = Σ nCk a^(n-k) b^k	129
General term	T_{r+1} = nCr a^(n-r) b^r	130
(x − y)^n	Σ (−1)^k nCk x^(n-k) y^k	130
(1 + x)^n	Σ nCk x^k	130
(1 − x)^n	Σ (−1)^k nCk x^k	131
Sum of coefficients	2^n	131
Alternating sum	0	131
Even-index sum	2^(n−1)	131
Odd-index sum	2^(n−1)	131
nC0	1	128
nCn	1	128
Pascal's identity	nCr + nC(r−1) = (n+1)Cr	129
Symmetry	nCr = nC(n−r)	117 (Ch. 6)
Middle term (even n)	T_{n/2+1}	130
Middle terms (odd n)	T_{(n+1)/2} and T_{(n+3)/2}	130
Coeff of x^k in (a+bx)^n	nCk a^(n-k) b^k	130
Numerical (98)^5	9039207968	131
6^n − 5n mod 25	1	132
Number of terms	n + 1	130
Sum of indices	n	130
Coefficient ratio T_{r+1}/T_r	(n−r+1)/r · b/a	130
Coefficient sign in (a−b)^n	Alternating	130
Expansion of (a+b)^7	8 terms	128

Formula	Statement	NCERT page
Highest-power term	a^n (when k = 0)	130
Lowest-power-of-a term	b^n (when k = n)	130

2.6 Solved examples (NCERT-grounded)

Example A (NCERT §7.2.1 worked, p. 129). Expand (x + 2)^6.

Step 1 — coefficients (Pascal's row 6): 1, 6, 15, 20, 15, 6, 1. Step 2 — assemble terms: T_1 = x^6, T_2 = 6x^5·2, T_3 = 15x^4·4, ⋯ Step 3 — simplify: x^6 + 12x^5 + 60x^4 + 160x^3 + 240x^2 + 192x + **64**. **Answer:** as above.

Example B (NCERT §7.2.2 worked, p. 130). Expand (x − 2y)^5.

Step 1 — apply (x + (−2y))^5: coefficients (1, 5, 10, 10, 5, 1). Step 2 — alternating signs from −2y: signs (+, −, +, −, +, −). Step 3 — simplify: x^5 − 10x^4 y + 40x^3 y^2 − 80x^2 y^3 + 80xy^4 − 32y^5. **Answer:** as above.

Example C (NCERT §7.2.2, p. 131). Compute (98)^5.

Step 1 — write 98 = 100 − 2: apply (100 − 2)^5. Step 2 — expand: 100^5 − 5·100^4·2 + 10·100^3·4 − 10·100^2·8 + 5·100·16 − 32. Step 3 — sum: 10000000000 − 1000000000 + 40000000 − 800000 + 8000 − 32 = **9039207968**.

Example D (NCERT Example 4, p. 132). Show 6^n − 5n leaves remainder 1 on division by 25.

Step 1 — expand (1 + 5)^n: 1 + 5n + 25·nC2 + 125·nC3 + ⋯ + 5^n. Step 2 — subtract 5n + 1: 6^n − 5n − 1 = 25(nC2 + 5·nC3 + ⋯ + 5^(n−2)). Step 3 — conclude: RHS divisible by 25 ⇒ 6^n − 5n ≡ 1 (mod 25). **QED.**

Example E (Finding a specific term). Find the coefficient of x^7 in the expansion of (1 + x)^10.

Step 1 — apply (1+x)^n formula: coefficient of x^k is nCk. Step 2 — substitute n = 10, k = 7: 10C7. Step 3 — evaluate: 10C7 = 10C3 = 120. **Answer: 120**.

🎯 Practice MCQs

📊 PYQ Alignment

This chapter is a consistent contributor on the CUET (UG) Mathematics paper — typically around 7–9 MCQs across the section across recent cycles — with questions clustered around (a) the structure of `(a + b)^n` (number of terms, sum of indices, sigma form), (b) identifying the `(r+1)` -th term `nCr a^(n-r) b^r` and reading off a specific coefficient, (c) the standard expansions $(1 + x)^n$ / $(1 - x)^n$ and the resulting identities $\Sigma\ nCr = 2^n$ and $\Sigma\ (-1)^r\ nCr = 0$, and (d) short numerical-approximation problems modelled on $(98)^5$, $(0.99)^5$, $(1.01)^{1000000}$ examples.