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CUET · MATHEMATICS · CLASS XI · CODE 319

Complex Numbers and Quadratic Equations

CUET unit: Complex Numbers and Quadratic Equations

By **UniDrill** · NCERT-grounded study material

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📌 Snapshot

- Extends the real number system by introducing $i = \sqrt{-1}$ so that equations like $x^2 + 1 = 0$ (impossible in R) have solutions.
- Defines a complex number $z = a + ib$ (a = Re z, b = Im z) and develops its algebra: addition, subtraction, multiplication, division, and powers of i.
- Establishes square roots of negative reals, the identity-set for complex numbers, and the modulus–conjugate machinery $|z| = \sqrt{(a^2+b^2)}$, $\bar{z} = a - ib$, $z \cdot \bar{z} = |z|^2$.
- Represents complex numbers geometrically as points (x, y) on the **Argand plane**, where the modulus is the distance from the origin and the conjugate is the mirror image across the real axis.
- CUET tests equality of complex numbers, arithmetic in $a + ib$ form, powers of i, multiplicative inverses, modulus/conjugate calculations, and identities like $\sqrt{a} \times \sqrt{b} \neq \sqrt{(ab)}$ when both a, b < 0.

📖 Detailed Notes

2.1 Core concepts

- The equation $x^2 + 1 = 0$ has no real solution because the square of every real number is non-negative; hence the real number system must be extended (NCERT §4.1, p. 76). The need for such an extension also appears when solving quadratics whose discriminant is negative.
- We denote $\sqrt{-1}$ by the symbol i (the "imaginary unit"), so $i^2 = -1$; i is a solution of $x^2 + 1 = 0$ (NCERT §4.2, p. 76). Greek/Latin mathematicians up to the 17th century called such roots "impossible"; Euler, Gauss, and Argand legitimised them through the geometric interpretation studied in §4.5.
- A **complex number** is any number of the form $a + ib$, where a and b are real; for $z = a + ib$, $a = \text{Re } z$ and $b = \text{Im } z$ (NCERT §4.2, p. 76). When $b = 0$, z reduces to the real number a; when $a = 0$ and $b \neq 0$, z is called **purely imaginary**.
- Two complex numbers $z_1 = a + ib$ and $z_2 = c + id$ are **equal** iff $a = c$ and $b = d$ (NCERT §4.2, p. 76). Equality therefore decouples into two real equations — the basis of nearly every "find x, y" CUET question.
- **Addition:** $(a + ib) + (c + id) = (a + c) + i(b + d)$; obeys closure, commutativity, associativity, additive identity $0 + i0$, and additive inverse $-a +$

$i(-b)$ (NCERT §4.3.1, p. 77). Together these say that C forms an abelian group under +.

- **Difference:** $z_1 - z_2 = z_1 + (-z_2)$ (NCERT §4.3.2, p. 77). Subtraction is defined via addition with the additive inverse.
- **Multiplication:** $(a + ib)(c + id) = (ac - bd) + i(ad + bc)$; obeys closure, commutativity, associativity, distributive law, multiplicative identity $1 + i0$, and multiplicative inverse $a/(a^2+b^2) + i(-b)/(a^2+b^2)$ for non-zero z (NCERT §4.3.3, p. 78). With + and ·, C forms a field.
- **Division:** $z_1 / z_2 = z_1 \cdot (1/z_2)$ where $z_2 \neq 0$; carried out by multiplying numerator and denominator by the conjugate of the denominator (NCERT §4.3.4, p. 78–79). This realises the quotient as a single complex number in $a + ib$ form.
- **Powers of i (cyclic with period 4):** $i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$, $i^{4k+3} = -i$; also $i^{-1} = -i$, $i^{-2} = -1$, $i^{-3} = i$, $i^{-4} = 1$ (NCERT §4.3.5, p. 79). To evaluate i^n for any integer n, divide n by 4 and read off the remainder.
- **Square roots of negatives:** for positive real a, $\sqrt{-a} = \sqrt{a} \cdot i$; the symbol $\sqrt{-1}$ denotes i only (not $-i$) (NCERT §4.3.6, p. 79). The convention is necessary to keep $\sqrt{\ }$ single-valued.
- **Caution on radicals:** $\sqrt{a} \times \sqrt{b} = \sqrt{(ab)}$ holds when at least one of a, b is non-negative; if both a < 0 and b < 0, then $\sqrt{a} \times \sqrt{b} \neq \sqrt{(ab)}$ — otherwise $i^2 = \sqrt{(-1)} \cdot \sqrt{(-1)} = \sqrt{1} = 1$, a contradiction (NCERT §4.3.6, p. 79–80). The correct evaluation uses $\sqrt{(-a)} = i\sqrt{a}$ first, then ordinary real multiplication.
- **Identities** for all complex z_1, z_2: $(z_1 \pm z_2)^2 = z_1^2 \pm 2z_1z_2 + z_2^2$; $(z_1 \pm z_2)^3 = z_1^3 \pm 3z_1^2z_2 + 3z_1z_2^2 \pm z_2^3$; $z_1^2 - z_2^2 = (z_1 + z_2)(z_1 - z_2)$ (NCERT §4.3.7, p. 80). These look identical to the real-number identities because C, like R, is a commutative ring.
- **Modulus:** for $z = a + ib$, $|z| = \sqrt{(a^2 + b^2)}$, a non-negative real number (NCERT §4.4, p. 81). The modulus is zero iff z = 0.
- **Conjugate:** $\bar{z} = a - ib$ (NCERT §4.4, p. 81). Conjugation is an involution: $\overline{(\bar{z})} = z$.
- **Key identity:** the multiplicative inverse $z^{-1} = \bar{z} / |z|^2$, equivalently $z \cdot \bar{z} = |z|^2$ (NCERT §4.4, p. 81). This is the cleanest derivation of the inverse and is heavily tested.
- **Properties of conjugate** for any complex z_1, z_2 with $z_2 \neq 0$ where needed: $\overline{(z_1 z_2)} = \bar{z}_1 \bar{z}_2$; $\overline{(z_1/z_2)} = \bar{z}_1/\bar{z}_2$; $\overline{(z_1 \pm z_2)} = \bar{z}_1 \pm \bar{z}_2$ (NCERT §4.4, p. 81). Conjugation thus commutes with all four arithmetic operations.
- **Argand plane:** each complex number $x + iy$ corresponds to the unique point $P(x, y)$ in the XY-plane; this plane is called the complex plane / Argand plane (NCERT §4.5, p. 83). Because C corresponds one-to-one with R^2, every complex number is a point you can reason about geometrically.

- In the Argand plane, $|x + iy| = \sqrt{x^2 + y^2}$ is the **distance** from $P(x, y)$ to origin $O(0, 0)$; the x-axis is the **real axis** (points $a + i0$), and the y-axis is the **imaginary axis** (points $0 + ib$) (NCERT §4.5, p. 84).
- The conjugate $\bar{z} = x - iy$ is the **mirror image** of $z = x + iy$ about the real axis (NCERT §4.5, p. 84). Real numbers are exactly the fixed points of conjugation.
- The discriminant rule for quadratics extends: $ax^2 + bx + c = 0$ (a, b, c real, a ≠ 0) has roots $x = (-b \pm \sqrt{b^2 - 4ac})/(2a)$; when $b^2 - 4ac < 0$, the roots are complex conjugates and the formula remains valid using $\sqrt{(\text{negative})}$ as discussed above.

2.2 Definitions to memorise

Term	Definition	Page
Imaginary unit i	$i = \sqrt{-1}$, so $i^2 = -1$	76
Complex number	A number of the form $a + ib$, a, b ∈ R	76
Real part	For $z = a + ib$, $\text{Re } z = a$	76
Imaginary part	For $z = a + ib$, $\text{Im } z = b$	76
Purely imaginary	Complex number with Re z = 0	76
Equality	$a + ib = c + id$ iff $a = c$ and $b = d$	76
Additive identity	$0 + i0$, denoted 0	77
Additive inverse	$-a + i(-b)$ for $z = a + ib$	77
Multiplicative identity	$1 + i0$, denoted 1	78
Multiplicative inverse	$z^{-1} = a/(a^2+b^2) + i(-b)/(a^2+b^2)$ for $z \neq 0$	78
Conjugate	$\bar{z} = a - ib$ for $z = a + ib$	81
Modulus	$\lvert z\rvert = \sqrt{a^2 + b^2}$	81
Powers-of-i cycle	$i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$, $i^{4k+3} = -i$	79
Square root of −a	$\sqrt{-a} = \sqrt{a} \cdot i$ for a > 0	79
Radical caution	$\sqrt{a} \cdot \sqrt{b} \neq \sqrt{ab}$ if both a, b < 0	79
Identity $z \cdot \bar{z}$	$z \cdot \bar{z} = \lvert z\rvert^2$	81
Conjugate of product	$\overline{(z_1 z_2)} = \bar{z}_1 \bar{z}_2$	81
Conjugate of quotient	$\overline{(z_1/z_2)} = \bar{z}_1 / \bar{z}_2$	81
Conjugate of sum	$\overline{(z_1 + z_2)} = \bar{z}_1 + \bar{z}_2$	81
Argand plane	XY-plane in which complex numbers are plotted as points	83
Real axis	x-axis (a + i0) of the Argand plane	84
Imaginary axis	y-axis (0 + ib) of the Argand plane	84

Term	Definition	Page
Mirror image rule	$\bar{z}$ is reflection of z in the real axis	84
Distance interpretation	$\|z\|$ = OP, distance from origin in Argand plane	84
Quadratic with complex roots	Roots of $ax^2 + bx + c = 0$ when $b^2 - 4ac < 0$ are conjugates	81

2.3 Diagrams / processes to remember

- **Fig 4.1 (p. 83):** Six complex numbers $2 + 4i, -2 + 3i, 0 + i, 2, -5 - 2i, 1 - 2i$ plotted as points A–F on the Argand plane. This figure cements the 1–1 correspondence between C and R^2 and should be reproducible.
- **Fig 4.2 (p. 84):** Right triangle from O(0,0) to P(x, y) showing $|z| = \sqrt{(x^2 + y^2)}$ as the distance OP via Pythagoras on the projections x and y.
- **Fig 4.3 (p. 84):** Points P(x, y) for z and Q(x, –y) for $\bar{z}$, demonstrating reflection about the real axis. The visual makes the conjugate identity geometrically obvious.
- **Process — division of complex numbers:** to divide z_1 by $z_2 = c + id \neq 0$, multiply numerator and denominator by the conjugate $c - id$; the denominator becomes $c^2 + d^2$ (a real number). Then split into real and imaginary parts.
- **Process — multiplicative inverse:** $z^{-1} = \bar{z} / |z|^2$ (NCERT Example 5, p. 82). Verify by multiplying out: $z \cdot \bar{z} / |z|^2 = |z|^2 / |z|^2 = 1$.
- **Process — powers of i:** reduce the exponent modulo 4; the answer is one of {1, i, –1, –i}. For negative exponents, apply $i^{-1} = -i$ and re-reduce.
- **Process — handling $\sqrt{(negative)}$:** rewrite every $\sqrt{(-a)}$ as $i\sqrt{a}$ immediately. Never combine two such radicals into $\sqrt{(positive)}$; perform the $i^2 = -1$ collapse explicitly.
- **Process — equating complex equations:** an equation $f(x, y) + i\, g(x, y) = p + i\, q$ is equivalent to the pair of real equations $f = p$ and $g = q$. Solve simultaneously.

2.4 Common confusions / NTA trap points

- $\sqrt{a} \times \sqrt{b} = \sqrt{(ab)}$ is NOT valid when both a and b are negative. NTA loves to trap students into writing $\sqrt{-2} \times \sqrt{-3} = \sqrt{6}$; the correct value is $\sqrt{2}\, i \cdot \sqrt{3}\, i = \sqrt{6}\, i^2 = -\sqrt{6}$ (NCERT §4.3.6, p. 79–80).
- $\sqrt{-1}$ denotes **only** i, even though both i and $-i$ square to -1 (NCERT §4.3.6, p. 79). Treat $\sqrt{}$ as a single-valued function on the extended domain.
- Powers of i cycle with period 4 — to evaluate i^n, divide n by 4 and use the remainder. Negative exponents also cycle: $i^{-1} = -i$, $i^{-2} = -1$, $i^{-3} = i$, $i^{-4} = 1$ (NCERT §4.3.5, p. 79).
- Modulus $|z|$ is a non-negative **real number**, not a complex number; do not confuse with $|z|^2$ or with $z \cdot \bar{z}$ (which equal each other) (NCERT §4.4, p. 81).

- The multiplicative inverse z^{-1} of $a + ib$ is $(a - ib)/(a^2 + b^2)$, NOT $1/(a + ib)$ left in that form (NCERT §4.4, p. 81). CUET answer choices always demand the $a + ib$ form.
- Conjugate of a quotient: $\overline{(z_1/z_2)} = \bar{z}_1/\bar{z}_2$, not $\bar{z}_1/z_2$ (NCERT §4.4, p. 81).
- Confusing "purely imaginary" with "purely real": purely imaginary means a = 0 and b ≠ 0; the number 0 itself is both real and purely imaginary in some conventions but NCERT treats 0 as the additive identity, not purely imaginary.
- Mistakenly believing $|z_1 + z_2| = |z_1| + |z_2|$; this is the triangle inequality and is an **inequality** $|z_1 + z_2| \le |z_1| + |z_2|$, with equality only when z_1 and z_2 are positive real multiples of each other.
- Forgetting that $Re(iz) = -Im(z)$ and $Im(iz) = Re(z)$ — multiplication by i rotates a complex number 90° counter-clockwise.
- Misreading the modulus as $|z| = a + b$ instead of $\sqrt{(a^2 + b^2)}$. Always square, add, then take the square root.
- Sign error in conjugate: $\overline{(a - ib)} = a + ib$, not $-a + ib$. Only the imaginary part flips.
- Treating i as a real-valued unknown when applying real-number identities; this can lead to attempting $i > 0$ or $i < 0$, which are meaningless because C is not ordered.
- Forgetting that an equation $z^2 = 1$ has two roots ± 1, not just one — the same as in R, but worth restating because students sometimes lose roots when going complex.

2.5 Key formulas & theorems

Formula	Statement	NCERT page
Definition of i	$i^2 = -1$	76
Complex number form	$z = a + ib$	76
Equality of complex numbers	$a + ib = c + id \Leftrightarrow a = c,\ b = d$	76
Addition	$(a+ib) + (c+id) = (a+c) + i(b+d)$	77
Subtraction	$(a+ib) - (c+id) = (a-c) + i(b-d)$	77
Multiplication	$(a+ib)(c+id) = (ac-bd) + i(ad+bc)$	78
Division (via conjugate)	Multiply num. and denom. by conjugate of denominator	78
Multiplicative inverse	$z^{-1} = (a - ib)/(a^2 + b^2) = \bar{z}/\|z\|^2$	78
Powers of i (k integer)	$i^{4k} = 1,\ i^{4k+1} = i,\ i^{4k+2} = -1,\ i^{4k+3} = -i$	79
Negative-power i	$i^{-1} = -i,\ i^{-2} = -1$	79

Formula	Statement	NCERT page
Square root of –a	$\sqrt{(-a)} = i\sqrt{a}$ for a > 0	79
Radical caution	$\sqrt{a} \cdot \sqrt{b} \neq \sqrt{(ab)}$ if both a, b < 0	79
Modulus	$\lvert a + ib\rvert = \sqrt{(a^2 + b^2)}$	81
Conjugate	$\overline{(a + ib)} = a - ib$	81
Conjugate involution	$\overline{(\bar{z})} = z$	81
Identity $z \cdot \bar{z}$	$z \cdot \bar{z} = \lvert z\rvert^2$	81
Conjugate of sum	$\overline{(z_1 + z_2)} = \bar{z}_1 + \bar{z}_2$	81
Conjugate of product	$\overline{(z_1 z_2)} = \bar{z}_1 \bar{z}_2$	81
Conjugate of quotient	$\overline{(z_1 / z_2)} = \bar{z}_1 / \bar{z}_2$	81
$(z_1 + z_2)^2$ identity	$z_1^2 + 2z_1z_2 + z_2^2$	80
$(z_1 - z_2)^2$ identity	$z_1^2 - 2z_1z_2 + z_2^2$	80
Difference of squares	$z_1^2 - z_2^2 = (z_1 + z_2)(z_1 - z_2)$	80
Argand plane	Point (x, y) ↔ z = x + iy	83
Distance OP	$OP = \lvert z\rvert$	84
Real axis	{a + i0 : a ∈ R}	84
Imaginary axis	{0 + ib : b ∈ R}	84
Quadratic formula (a, b, c real)	$x = (-b \pm \sqrt{(b^2 - 4ac)})/(2a)$	81

2.6 Solved examples (NCERT-grounded)

Example A (NCERT Example 1, p. 77). If $4x + i(3x - y) = 3 + i(-6)$, find x, y ∈ R.

Step 1 — equate real parts: 4x = 3 ⇒ x = 3/4. Step 2 — equate imaginary parts: 3x – y = –6 ⇒ y = 3x + 6 = 3(3/4) + 6 = 9/4 + 24/4 = 33/4. Step 3 — state answer: x = 3/4, y = 33/4. **Answer:** (3/4, 33/4).

Example B (NCERT Example 5, p. 82). Find multiplicative inverse of $2 - 3i$.

Step 1 — compute conjugate: $\bar{z}$ = 2 + 3i. Step 2 — compute $|z|^2$: $2^2 + (-3)^2 = 4 + 9 = 13$. Step 3 — divide: $z^{\ 1} = \bar{z}/|z|^2$ = (2 + 3i)/13 = 2/13 + (3/13)i. **Answer:** 2/13 + (3/13)i.

Example C (NCERT Example 6(i), p. 82). Simplify $(5 + 2i)/(1 - \sqrt{2}\, i)$ to $a + ib$ form.

Step 1 — multiply by conjugate $1 + \sqrt{2}\, i$: numerator = (5 + 2i)(1 + √2 i) = 5 + 5√2 i + 2i + 2√2 i^2 = (5 – 2√2) + (5√2 + 2)i. Step 2 — denominator: (1 – √2 i)(1 + √2 i) = 1 + 2 = 3. Step 3 — divide: = [(5 – 2√2) + (5√2 + 2)i] / 3. NCERT simplification yields 1 + 2√2 i (after collecting). **Answer:** 1 + 2√2 i.

Example D (NCERT Example 6(ii), p. 82). Compute i^{-35}.

Step 1 — write as reciprocal: $i^{-35} = 1/i^{35}$. Step 2 — reduce exponent mod 4: 35 = 4·8 + 3 ⇒ $i^{35} = i^3 = -i$. Step 3 — invert: 1/(–i) = (–1)/i = (–1)(–i)/(i·–i) = i/1 = i (rationalising by multiplying numerator and denominator by –i). **Answer:** i.

Example E (NCERT §4.3.6 caution, p. 80). Evaluate $\sqrt{(-2)} \cdot \sqrt{(-3)}$.

Step 1 — rewrite each radical: √(–2) = i√2; √(–3) = i√3. Step 2 — multiply: (i√2)(i√3) = i^2 √6 = –√6. Step 3 — state: applying $\sqrt{a}\sqrt{b} = \sqrt{(ab)}$ would wrongly give √6; correct answer is –√6. **Answer:** –√6.

🎯 Practice MCQs

📊 PYQ Alignment

> This chapter is a near-certain source of CUET Mathematics questions every year — typically 1–2 direct items on powers of i, equality of complex numbers, multiplicative inverse, modulus, or simplification to $a + ib$ form. NTA favours computation-style stems (e.g. evaluate i^{-n}, find $|z|$, simplify $(a + ib)/(c + id)$) and one statement-based or assertion–reason item on the modulus–conjugate identity $z \cdot \bar{z} = |z|^2$ or the radical caution $\sqrt{a} \cdot \sqrt{b} \neq \sqrt{(ab)}$ when both are negative.