FREE EDITION · NOTES + 3 SAMPLE MCQS

CUET · MATHEMATICS · CLASS XI · CODE 319

Conic Sections

CUET unit: Conic Sections

By **UniDrill** · NCERT-grounded study material

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📌 Snapshot

- Establishes conics (circle, ellipse, parabola, hyperbola) as plane sections of a double-napped right circular cone and classifies them by the angle β the cutting plane makes with the axis relative to the semi-vertical angle α.
- Develops the four standard equations (circle, parabola, ellipse, hyperbola) with centre/vertex at the origin and axes along the coordinate axes.
- The focus–directrix and focus–sum/focus–difference definitions yield the focal-distance and eccentricity relationships.
- Defines latus rectum for parabola (4a), ellipse and hyperbola ($2b^2/a$) and uses them in identification problems.
- Closes with degenerated conics (point, line, pair of intersecting lines) obtained when the plane passes through the vertex of the cone — useful for CUET assertion-reason traps.

📖 Detailed Notes

2.1 Core concepts

- A **conic section** is a curve obtained by intersecting a right circular cone (double-napped) with a plane; the type depends on the angle β the plane makes with the vertical axis and the semi-vertical angle α of the cone (NCERT §10.2, p. 176–177). The Greek mathematician Apollonius (~200 BCE) first studied these systematically.
- **Sections (non-degenerate, plane not through vertex):** $\beta = 90^\circ \rightarrow$ **circle**; $\alpha < \beta < 90^\circ \rightarrow$ **ellipse**; $\beta = \alpha \rightarrow$ **parabola**; $0 \leqslant \beta < \alpha \rightarrow$ **hyperbola** (plane cuts both nappes) (NCERT §10.2.1, p. 177).
- **Degenerated conics (plane through vertex):** $\alpha < \beta \leqslant 90^\circ \rightarrow$ a **point**; $\beta = \alpha \rightarrow$ a **straight line** (degenerate parabola); $0 \leqslant \beta < \alpha \rightarrow$ a **pair of intersecting straight lines** (degenerate hyperbola) (NCERT §10.2.2, p. 178). These correspond to "limit" cases when the plane just touches the vertex.
- **Circle:** locus of points in a plane equidistant from a fixed point (centre); equation with centre (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$; with centre at origin it reduces to $x^2 + y^2 = r^2$ (NCERT §10.3, p. 179–180). Eccentricity of circle is 0 (a limiting case of ellipse).
- The general form $x^2 + y^2 + 8x + 10y - 8 = 0$ is reduced by completing the square to $(x + 4)^2 + (y + 5)^2 = 49$, giving centre (–4, –5) and radius 7 — the standard NCERT

technique for any general circle equation (NCERT Example 3, p. 180). Any equation $Ax^2 + Ay^2 + Bx + Cy + D = 0$ ($A \neq 0$) represents a circle.

- **Parabola:** set of all points in a plane equidistant from a fixed line (**directrix**) and a fixed point (**focus**) not on the line; the line through the focus perpendicular to the directrix is the **axis**; intersection of parabola with the axis is the **vertex** (NCERT §10.4, p. 182). Eccentricity of parabola is 1.
- **Four standard parabolas** (vertex at origin, axis on a coordinate axis): $y^2 = 4ax$ (opens right), $y^2 = -4ax$ (opens left), $x^2 = 4ay$ (opens upward), $x^2 = -4ay$ (opens downward); a y^2 term ⇒ axis along x-axis, an x^2 term ⇒ axis along y-axis (NCERT §10.4.1, p. 183–184).
- **Latus rectum** of a parabola is the chord through the focus perpendicular to the axis with endpoints on the curve; its length for $y^2 = 4ax$ is **4a** (NCERT §10.4.2, p. 185).
- **Ellipse:** set of points the sum of whose distances from two fixed points (**foci**) is a constant 2a, with 2a greater than the distance 2c between foci (NCERT §10.5, p. 187).
- **Ellipse geometry:** length of major axis 2a, minor axis 2b, distance between foci 2c, and the relation $c^2 = a^2 - b^2$ (equivalently $a^2 = b^2 + c^2$) (NCERT §10.5.1, p. 188). The square-root relation arises from the right triangle B–C–F where B is on the minor axis.
- **Eccentricity of ellipse:** $e = c/a$; since $c < a$, we have $e < 1$; focus lies at distance ae from the centre (NCERT §10.5.2, p. 188). The closer e is to 0, the more circular; the closer to 1, the more elongated.
- **Standard equations of ellipse:** with centre at origin, $x^2/a^2 + y^2/b^2 = 1$ (foci on x-axis) and $x^2/b^2 + y^2/a^2 = 1$ (foci on y-axis); the foci always lie on the major axis — i.e., on the axis whose square has the **larger denominator** (NCERT §10.5.3, p. 189–191).
- **Latus rectum of ellipse:** the chord through a focus perpendicular to the major axis with endpoints on the ellipse; its length is $2b^2/a$ (NCERT §10.5.4, p. 192).
- **Hyperbola:** set of points the **difference** of whose distances from two fixed foci is a constant 2a; mid-point of foci = centre; line through foci = **transverse axis**; perpendicular through centre = **conjugate axis**; intersections with transverse axis = **vertices** (NCERT §10.6, p. 195–196).
- **Hyperbola geometry:** distance between foci 2c, length of transverse axis 2a, length of conjugate axis 2b, with $b = \sqrt{(c^2 - a^2)}$ so that $c^2 = a^2 + b^2$ (NCERT §10.6, p. 196). Note the sign difference from the ellipse relation.
- **Eccentricity of hyperbola:** $e = c/a$; since $c \geqslant a$ we always have $e \geqslant 1$; foci are at distance ae from centre (NCERT §10.6.1, p. 197).

- **Standard equations of hyperbola** (centre at origin): $x^2/a^2 - y^2/b^2 = 1$ (transverse axis along x-axis) and $y^2/a^2 - x^2/b^2 = 1$ (transverse axis along y-axis); the **positive term's denominator** gives the transverse axis (NCERT §10.6.2, p. 197–199).
- An **equilateral hyperbola** is one in which a = b (NCERT §10.6.2 Note, p. 199). Its eccentricity is $\sqrt{2}$.
- **Latus rectum of hyperbola:** chord through a focus perpendicular to the transverse axis; its length is **$2b^2/a$** — identical formula to the ellipse (NCERT §10.6.3, p. 200).
- The four conics interrelate: circle is a special ellipse with e = 0 (a = b); parabola is the limit of an ellipse (or hyperbola) as $e \to 1$; hyperbola has $e > 1$. The unified focus–directrix definition gives all four as "loci of constant ratio of distances", with the ratio e determining the type.
- Asymptotes of the standard hyperbola $x^2/a^2 - y^2/b^2 = 1$ are the lines $y = \pm(b/a)\,x$; these are the diagonals of the central rectangle of sides 2a × 2b and serve as guidelines for sketching the hyperbola accurately.
- Practical curve-sketching: for the ellipse, mark vertices (±a, 0), co-vertices (0, ±b), foci (±c, 0); for the hyperbola, mark vertices (±a, 0), foci (±c, 0), and asymptotes; for the parabola, mark vertex (0, 0), focus (a, 0), and directrix x = –a.
- Every conic can be written in polar coordinates centred at a focus as $r = ed/(1 + e \cos \theta)$, where d is the distance from focus to directrix; this unified treatment is studied in advanced courses.
- Conic sections appear extensively in physics (Kepler orbits — ellipses, parabolic trajectories, hyperbolic comet paths) and engineering (parabolic mirrors, elliptical gears).
- Quick-recognition tip for CUET MCQs: see $y^2 =$ ___ x ⇒ parabola opening along x-axis; $x^2 + y^2 =$ ___ ⇒ circle; positive coefficients on both x^2 and y^2 with unequal denominators ⇒ ellipse; one positive, one negative ⇒ hyperbola.
- Important historical note: Pappus of Alexandria proved (around 320 CE) the unified focus–directrix theorem; Kepler used the ellipse in his first law of planetary motion (1609); Newton's gravitational theory rigorously derives Keplerian orbits as conics.

2.2 Definitions to memorise

Term	Definition	Page
Conic section	Plane × cone curve	177
Circle	Equidistant from centre	179
Parabola	Equidistant from focus and directrix	182
Ellipse	Constant sum to two foci = 2a	187
Hyperbola	Constant difference of distances to two foci = 2a	195

Term	Definition	Page
Focus	Fixed point in focus-directrix or focus-sum definition	182
Directrix	Fixed line for parabola	182
Axis of parabola	Perpendicular line through focus	182
Vertex (parabola)	Intersection with axis	182
Major axis (ellipse)	Length 2a, contains foci	187
Minor axis (ellipse)	Length 2b, perpendicular to major	187
Transverse axis (hyperbola)	Contains foci, length 2a	196
Conjugate axis (hyperbola)	Perpendicular at centre, length 2b	196
Eccentricity	$e = c/a$	188
Eccentricity bounds	Ellipse $e < 1$; parabola $e = 1$; hyperbola $e > 1$	188, 197
c^2 (ellipse)	$a^2 - b^2$	188
c^2 (hyperbola)	$a^2 + b^2$	196
Latus rectum (parabola)	4a	185
Latus rectum (ellipse)	$2b^2/a$	192
Latus rectum (hyperbola)	$2b^2/a$	200
Degenerated parabola	Single straight line	178
Degenerated hyperbola	Pair of intersecting lines	178
Equilateral hyperbola	$a = b$	199
Standard circle	$x^2 + y^2 = r^2$	180
Standard parabola	$y^2 = 4ax$ (and variants)	183
Standard ellipse	$x^2/a^2 + y^2/b^2 = 1$	189
Standard hyperbola	$x^2/a^2 - y^2/b^2 = 1$	197

2.3 Diagrams / processes to remember

- Fig 10.1–10.3 (p. 176–177): line m rotated about axis l at fixed angle α generates the double-napped cone; β is the angle the cutting plane makes with the vertical axis.
- Fig 10.4–10.7 (p. 178): the four non-degenerate cases — circle ($\beta = 90°$), ellipse ($\alpha < \beta < 90°$), parabola ($\beta = \alpha$), hyperbola ($0 \leqslant \beta < \alpha$, cuts both nappes).
- Fig 10.8–10.10 (p. 179): degenerated conics — point, single line, pair of intersecting straight lines (plane through vertex).
- Fig 10.11–10.12 (p. 180): circle centred at origin and at (h, k).
- Fig 10.15 (a)–(d) (p. 183): the four standard parabolas $y^2 = 4ax$, $y^2 = -4ax$, $x^2 = 4ay$, $x^2 = -4ay$.

- Fig 10.17–10.18 (p. 185): construction showing the latus rectum length 4a for $y^2 = 4ax$.
- Fig 10.21–10.23 (p. 187–188): ellipse — vertices, major/minor axis, the right-triangle picture that yields $a^2 = b^2 + c^2$.
- Fig 10.27–10.30 (p. 196–198): hyperbola — vertices on transverse axis, $b = \sqrt{(c^2 - a^2)}$, two standard orientations.
- **Process — reduce general circle to standard form:** group x-terms and y-terms; complete squares; rearrange to $(x - h)^2 + (y - k)^2 = r^2$ form; read off centre and radius.
- **Process — identify parabola:** check which variable is squared ($y^2 \Rightarrow$ horizontal axis; $x^2 \Rightarrow$ vertical axis); check sign of the linear coefficient (positive ⇒ opens right/up; negative ⇒ left/down); compute a = (|coefficient|)/4 and locate focus at (a, 0) or (0, a).
- **Process — find focus/eccentricity of ellipse:** read a^2, b^2 from denominators ($a > b$); compute $c = \sqrt{(a^2 - b^2)}$; foci at (±c, 0) if $x^2/a^2 + y^2/b^2$ with $a > b$; $e = c/a$; latus rectum = $2b^2/a$.
- **Process — find focus/eccentricity of hyperbola:** read a^2 (positive term), b^2 (negative term); compute $c = \sqrt{(a^2 + b^2)}$; $e = c/a$; latus rectum = $2b^2/a$.

2.4 Common confusions / NTA trap points

- **c^2 sign flip:** for an ellipse $c^2 = a^2 - b^2$ (so $b < a$ along major axis), but for a hyperbola $c^2 = a^2 + b^2$. Mixing the two is the most common slip.
- **Which axis carries the foci?** For an ellipse the foci lie on the **major axis** — i.e., the axis whose square has the **larger denominator** (p. 191). For a hyperbola the foci lie on the **transverse axis** — i.e., the axis of the **positive term** in $x^2/a^2 - y^2/b^2 = 1$ (p. 200). NTA distractors swap these rules.
- **Direction of opening for parabola:** $y^2 = 4ax$ opens **right**, $y^2 = -4ax$ opens **left**, $x^2 = 4ay$ opens **up**, $x^2 = -4ay$ opens **down** (p. 184). Misreading the sign or coordinate puts foci on the wrong axis.
- **Eccentricity bounds:** $e < 1$ for ellipse, $e = 1$ conceptually corresponds to the parabola case ($\beta = \alpha$), $e > 1$ for hyperbola. Confusing "$e \leqslant 1$" with "$e < 1$" is a trap.
- **Latus-rectum formula reuse:** $2b^2/a$ is the latus-rectum length for **both** ellipse and hyperbola — but the underlying b^2 differs because $c^2 = a^2 - b^2$ vs $c^2 = a^2 + b^2$. Plugging the wrong relation gives a wrong b^2 and a wrong latus rectum.
- **Confusing focus and directrix:** the focus is a point, the directrix is a line. The parabola is the locus equidistant from a point and a line; ellipse and hyperbola are loci involving distances to two points.
- **Vertex vs centre:** the parabola has a single vertex; the ellipse and hyperbola each have two vertices and one centre.

- **Forgetting to compare a and b for ellipse:** if $a < b$ in the equation $x^2/a^2 + y^2/b^2 = 1$, then the major axis is along the y-axis, not the x-axis.
- **Confusing 4a (parabola) with $2b^2/a$ (ellipse/hyperbola):** the latus rectum formula differs by type.
- **Mis-identifying the type from a general equation:** look for $Ax^2 + Cy^2 + \cdots$; $A = C \Rightarrow$ circle (after appropriate sign); A, C same sign different magnitudes ⇒ ellipse; A, C opposite signs ⇒ hyperbola; one of A, C zero ⇒ parabola.
- **Mistreating a^2, b^2 as algebraic quantities that can be negative:** they are always positive squares.
- **Confusing the focal chord with latus rectum:** latus rectum is the specific focal chord perpendicular to the axis.

2.5 Key formulas & theorems

Formula	Statement	NCERT page
Circle (general)	$(x - h)^2 + (y - k)^2 = r^2$	180
Circle (origin)	$x^2 + y^2 = r^2$	180
General to standard	Complete the square	180
Parabola $y^2 = 4ax$	Focus $(a, 0)$; directrix $x = -a$	183
Parabola $y^2 = -4ax$	Focus $(-a, 0)$; directrix $x = a$	184
Parabola $x^2 = 4ay$	Focus $(0, a)$; directrix $y = -a$	184
Parabola $x^2 = -4ay$	Focus $(0, -a)$; directrix $y = a$	184
Latus rectum (parabola)	$4a$	185
Ellipse $x^2/a^2 + y^2/b^2 = 1$ $(a > b)$	Foci $(\pm c, 0)$; $c^2 = a^2 - b^2$	189
Ellipse $x^2/b^2 + y^2/a^2 = 1$ $(a > b)$	Foci $(0, \pm c)$; $c^2 = a^2 - b^2$	191
Eccentricity (ellipse)	$e = c/a < 1$	188
Latus rectum (ellipse)	$2b^2/a$	192
Hyperbola $x^2/a^2 - y^2/b^2 = 1$	Foci $(\pm c, 0)$; $c^2 = a^2 + b^2$	197
Hyperbola $y^2/a^2 - x^2/b^2 = 1$	Foci $(0, \pm c)$; $c^2 = a^2 + b^2$	199
Eccentricity (hyperbola)	$e = c/a \geqslant 1$	197
Latus rectum (hyperbola)	$2b^2/a$	200
Equilateral hyperbola	$a = b \Rightarrow e = \sqrt{2}$	199
Eccentricity (circle)	0	188
Eccentricity (parabola)	1	188
Major axis length	$2a$	187
Minor axis length	$2b$	187

Formula	Statement	NCERT page
Transverse axis length	2a	196
Conjugate axis length	2b	196
Distance between foci	2c	187
Vertex of $y^2 = 4ax$	(0, 0)	183
Asymptotes (hyperbola)	$y = \pm(b/a)\,x$	198

2.6 Solved examples (NCERT-grounded)

Example A (NCERT Example 3, p. 180). Centre, radius of $x^2 + y^2 + 8x + 10y - 8 = 0$.

Step 1 — complete squares: $(x + 4)^2 - 16 + (y + 5)^2 - 25 - 8 = 0$. Step 2 — rearrange: $(x + 4)^2 + (y + 5)^2 = 49$. Step 3 — read off: **centre (−4, −5), radius 7**.

Example B (NCERT Example 5, p. 185). Focus and latus rectum of $y^2 = 8x$.

Step 1 — compare with $y^2 = 4ax$: $4a = 8 \Rightarrow a = 2$. Step 2 — focus: (a, 0) = **(2, 0)**. Step 3 — latus rectum: 4a = **8**.

Example C (NCERT Example 7, p. 186). Equation of parabola with vertex (0,0), focus (0, 2).

Step 1 — focus on y-axis: axis along y-axis; form $x^2 = 4ay$ with a = 2. Step 2 — substitute: $x^2 = 4 \cdot 2 \cdot y$. Step 3 — final: $\mathbf{x^2 = 8y}$.

Example D (NCERT Example 9, p. 192). Eccentricity, latus rectum of $x^2/25 + y^2/9 = 1$.

Step 1 — read a, b: a = 5, b = 3. Step 2 — $c = \sqrt{(a^2 - b^2)}$: $\sqrt{(25 - 9)} = 4$. Step 3 — e and LR: e = c/a = **4/5**; $LR = 2b^2/a = 18/5$.

Example E (NCERT Example 14(i), p. 200). Foci, e, LR of $x^2/9 - y^2/16 = 1$.

Step 1 — a, b: a = 3, b = 4. Step 2 — c: $\sqrt{(9 + 16)} = 5$. Step 3 — read off: foci **(±5, 0)**, e = 5/3, LR = 32/3.

🎯 Practice MCQs

📊 PYQ Alignment

> Conic Sections has been a steady scorer on CUET (UG) Mathematics, with roughly 8–10 MCQs annually drawn from identification of the conic type, reading off focus / vertex / directrix / latus rectum from standard equations, applying $c^2 = a^2 \mp b^2$ for ellipse and

hyperbola, and reducing general circle equations to standard form by completing the square — all squarely within the NCERT chapter scope above.