FREE EDITION · NOTES + 3 SAMPLE MCQS

CUET · MATHEMATICS · CLASS XI · CODE 319

Introduction to Three Dimensional Geometry

CUET unit: Introduction to Three Dimensional Geometry

By UniDrill · NCERT-grounded study material

📌 Snapshot

- Extends 2-D Cartesian geometry to 3-D space: a point now needs **three** coordinates (x, y, z) measured as perpendicular distances from three mutually perpendicular coordinate planes.
- Establishes the rectangular coordinate system: three axes (X′OX, Y′OY, Z′OZ), three coordinate planes (XY, YZ, ZX), and the resulting eight octants.
- Develops the **distance formula** in 3-D: $PQ = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$, used to test collinearity, identify triangles (isosceles / right-angled), and derive loci.
- CUET typically lifts direct numerical applications of the distance formula and octant-identification questions straight from this chapter.

📖 Detailed Notes

2.1 Core concepts

- A point in a plane is fixed by two perpendicular coordinate axes, but a point in space requires **three** numbers — perpendicular distances from three mutually perpendicular planes. NCERT uses the floor-and-two-adjacent-walls analogy: any object inside a room can be located by giving its perpendicular distances from the floor and the two walls (NCERT §11.1, p. 208). This intuition is the conceptual bridge from 2-D (where two numbers suffice) to 3-D (where three numbers are necessary).
- In 3-D space, three planes intersecting at point O, mutually perpendicular, give three lines X′OX, Y′OY, Z′OZ called the **x-, y-, z-axes** respectively; together they form the **rectangular coordinate system** (NCERT §11.2, p. 209). The choice of which two axes to call "horizontal" and which to call "vertical" is a convention — NCERT takes the XOY plane as the plane of paper and Z′OZ as the vertical line.
- The three coordinate planes are the **XY-plane (XOY), YZ-plane (YOZ), and ZX-plane (ZOX)**. Each pair of axes determines exactly one coordinate plane, and the three planes mutually intersect along the three axes at the origin O (NCERT §11.2, p. 209).
- Sign convention: above the XY-plane (along OZ) is positive, below (along OZ′) negative; right of the ZX-plane (along OY) positive, left (along OY′) negative; in front of the YZ-plane (along OX) positive, behind (along OX′) negative (NCERT §11.2, p.

209). This convention is invariant across all chapters of Class XII vectors and 3-D geometry, so memorising it now pays compound dividends.

- The point O is the **origin**, and the three coordinate planes divide space into **eight octants**, denoted I through VIII (NCERT §11.2, p. 209). Octant I is the all-positive octant (+, +, +); the remaining seven octants flip one, two, or all three signs.
- Given a point P in space, drop perpendicular PM onto the XY-plane; from M drop perpendicular ML to the x-axis. Then OL = x, LM = y, MP = z are the x-, y-, z-coordinates of P (NCERT §11.3, p. 209). This "drop two perpendiculars" recipe is the constructive definition NCERT uses, and it is the cleanest way to visualise coordinates without resorting to vectors.
- Equivalently, through P draw three planes parallel to the coordinate planes meeting the x-, y-, z-axes at A, B, C with OA = x, OB = y, OC = z. Hence x, y, z are perpendicular distances from the YZ-, ZX-, XY-planes respectively (NCERT §11.3, p. 210). This is the formulation most often tested by CUET: x measures distance from YZ-plane, y from ZX-plane, z from XY-plane.
- Special points: origin O = (0, 0, 0); any point on the x-axis has form (x, 0, 0); any point in the YZ-plane has form (0, y, z) (NCERT §11.3, Note, p. 210). By symmetry, points on the y-axis are (0, y, 0), on the z-axis (0, 0, z), in the XY-plane (x, y, 0), and in the ZX-plane (x, 0, z).
- The signs of (x, y, z) determine the octant uniquely; Table 11.1 lists the sign combination for all eight octants (NCERT §11.3, Remark and Table 11.1, p. 210). For example, octant I = (+, +, +), II = (–, +, +), III = (–, –, +), IV = (+, –, +), V = (+, +, –), VI = (–, +, –), VII = (–, –, –), VIII = (+, –, –).
- **Distance formula in 3-D:** for $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$, drawing planes through P and Q parallel to coordinate planes forms a rectangular parallelopiped with diagonal PQ. Using right angles at A (in △PAQ) and at N (in △ANQ): $PQ^2 = PA^2 + AN^2 + NQ^2 = (x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2$. Therefore $PQ = \sqrt{[(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2]}$ (NCERT §11.4, pp. 211–212). The 2-D distance formula $PQ = \sqrt{[(x_2-x_1)^2 + (y_2-y_1)^2]}$ is the special case $z_1 = z_2 = 0$.
- **Distance from origin:** if P = O = (0, 0, 0), then $OQ = \sqrt{(x_2^2 + y_2^2 + z_2^2)}$ (NCERT §11.4, p. 212). This is the modulus of the position vector of Q (anticipating Class XII vector algebra).
- Worked applications: distance between two given points (Example 3, p. 212); proving three points are collinear by showing PQ + QR = PR (Example 4, p. 212); checking whether three points are vertices of a right-angled triangle by Pythagoras on the three squared sides (Example 5, pp. 212–213); deriving the equation of a locus from a distance condition such as $PA^2 + PB^2 = 2k^2$ (Example 6, p. 213). Each of these problem types reappears almost verbatim on CUET.
- Miscellaneous results derived using distance formula: showing four points form a parallelogram but not a rectangle by computing sides and diagonals (Example 7, p. 214); locus of points equidistant from two given points yields a plane (Example 8, p.

214); using the centroid formula $G = ((x_1+x_2+x_3)/3, (y_1+y_2+y_3)/3, (z_1+z_2+z_3)/3)$ to find an unknown vertex when the centroid is known (Example 9, pp. 214–215).

- Scope here is limited to coordinates and distance; the section formula, direction cosines, and equations of lines/planes are deferred to Class XII (NCERT §11.4 closing remark, p. 215).

2.2 Definitions to memorise

Term	Definition	Page
Coordinate axes	Three mutually perpendicular lines X′OX, Y′OY, Z′OZ intersecting at origin O	209
Coordinate planes	The three planes XY (XOY), YZ (YOZ), ZX (ZOX) determined by pairs of axes	209
Origin	The common point O of the three coordinate axes; coordinates (0, 0, 0)	209–210
Octants	The eight parts into which the three coordinate planes divide space	209
XY-plane	Plane containing the x- and y-axes; every point on it has z = 0	209
YZ-plane	Plane containing the y- and z-axes; every point on it has x = 0	209
ZX-plane	Plane containing the z- and x-axes; every point on it has y = 0	209
Coordinates of P	Ordered triplet (x, y, z) giving perpendicular distances from YZ-, ZX-, XY-planes	210
x-coordinate	Perpendicular distance of P from the YZ-plane	210
y-coordinate	Perpendicular distance of P from the ZX-plane	210
z-coordinate	Perpendicular distance of P from the XY-plane	210
Point on x-axis	Has form (x, 0, 0)	210
Point on y-axis	Has form (0, y, 0)	210
Point on z-axis	Has form (0, 0, z)	210
Point in XY-plane	Has form (x, y, 0)	210
Octant I	Sign pattern (+, +, +)	210
Octant V	Sign pattern (+, +, –)	210
Distance formula	$PQ = \sqrt{[(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2]}$	212
Distance from origin	$OQ = \sqrt{(x_2^2 + y_2^2 + z_2^2)}$ for $Q(x_2, y_2, z_2)$	212
Collinear points	Points P, Q, R lying on one line; verified when PQ + QR = PR	212

Term	Definition	Page
Centroid of triangle	$G = ((x_1+x_2+x_3)/3, (y_1+y_2+y_3)/3, (z_1+z_2+z_3)/3)$	214
Rectangular parallelopiped	Box used to derive distance formula via diagonal PQ	211
Right-angled triangle test	Verify Pythagoras $a^2 + b^2 = c^2$ on the three squared sides	213
Locus from distance condition	Equation in x, y, z obtained from $PA^2 + PB^2$ = constant	213

2.3 Diagrams / processes to remember

- **Fig 11.1 (p. 209):** Three coordinate planes intersecting at O with axes X′OX, Y′OY, Z′OZ — fixes orientation and sign convention. The plane of the page is taken to be the XOY plane, with Z′OZ piercing the page vertically. Reproduce this figure from memory, since every later 3-D figure builds on it.
- **Fig 11.2 (p. 209):** Construction of (x, y, z) for a point P — drop perpendicular PM to the XY-plane (M is the foot of perpendicular), then from M drop perpendicular ML to the x-axis. Read OL = x, LM = y, MP = z. The diagram is the cleanest geometric demonstration that the three coordinates are independent perpendicular measurements.
- **Fig 11.3 (p. 210):** Alternative "three parallel planes" construction — through P draw three planes parallel to the YZ-, ZX-, and XY-planes; these meet the x-, y-, z-axes at A, B, C with OA = x, OB = y, OC = z. This is the picture you should sketch when asked to interpret a coordinate as a "perpendicular distance from a plane".
- **Table 11.1 (p. 210):** Sign pattern of (x, y, z) in each of the eight octants I–VIII — essential for octant-identification MCQs. Memorise the cycle: octants I–IV have $z > 0$ (above the XY-plane), octants V–VIII have $z < 0$ (below). Within each band of four, the x-y sign pattern rotates anti-clockwise starting from (+, +).
- **Fig 11.4 (p. 211):** Rectangular parallelopiped with diagonal PQ used to derive the distance formula via two right-angled triangles (△PAQ and △ANQ). The edges of the box are $|x_2-x_1|$, $|y_2-y_1|$, $|z_2-z_1|$, and the diagonal is PQ. This is the 3-D analogue of the 2-D "right triangle on the difference of coordinates" picture used to derive $\sqrt{((x_2-x_1)^2 + (y_2-y_1)^2)}$.
- **Process — distance computation:** (i) subtract corresponding coordinates, (ii) square each difference, (iii) sum the three squares, (iv) take the positive square root. Algebraic sign of the differences is irrelevant because of the squaring; this is a common student worry.
- **Process — collinearity test:** compute PQ, QR, PR and check whether the largest equals the sum of the other two. If yes, P, Q, R are collinear (NCERT §11.4, Example 4, p. 212).

- **Process — triangle type test:** compute the three squared sides a^2, b^2, c^2 with c the largest. Right-angled iff $a^2 + b^2 = c^2$; isosceles iff exactly two of a^2, b^2, c^2 are equal; equilateral iff all three are equal.

2.4 Common confusions / NTA trap points

- Confusing which coordinate is the distance from which plane: x is the perpendicular distance from the **YZ-plane** (not the x-axis), y from the ZX-plane, z from the XY-plane (NCERT §11.3, p. 210). The mnemonic: "the coordinate equals the distance from the plane that does NOT bear its letter".
- Mis-naming octants from Table 11.1: e.g. point (–3, 1, 2) lies in octant **II** (not III), and (–3, 1, –2) lies in octant **VI** (Example 2, p. 211). The trap is reading the z-sign first instead of the x-sign.
- Forgetting that any point in the XZ-plane has y = 0, and any point on the x-axis has both y = 0 and z = 0 (Exercise 11.1 Q1, Q2, p. 211). MCQs often phrase this as "the locus of points with y = 0 is ⋯".
- Forgetting to square each coordinate difference inside the radical, or to take the **positive** square root for distance. Distance is always non-negative.
- Concluding "right-angled triangle" without verifying Pythagoras: in Example 5, $CA^2 + AB^2 \neq BC^2$, so △ABC is **not** right-angled even though it looks plausible (p. 213). Always compute all three squared sides and test all three combinations.
- Reading (x, y, z) in the wrong order — coordinates are an ordered triplet, so (1, 2, 3) ≠ (3, 2, 1).
- Forgetting that distance is symmetric: PQ = QP. Some students recompute QP and get a sign error.
- Mistaking the 2-D distance formula for 3-D and dropping the $(z_2-z_1)^2$ term — a frequent calculation slip under time pressure.
- Confusing "octant" with "quadrant": there are four quadrants in 2-D and eight octants in 3-D.
- Writing coordinates of a point on the y-axis as (y, 0, 0) instead of (0, y, 0) — the only non-zero entry is the second.
- Thinking that the equation of the XY-plane is x = 0 or y = 0; it is actually z = 0 (the XY-plane is the set of points with zero z-coordinate).
- Forgetting that the centroid formula in 3-D is the direct extension of 2-D — all three coordinates are averaged independently (NCERT §11.4, Example 9, p. 214).

2.5 Key formulas & theorems

Formula	Statement	NCERT page
Coordinates of P	P = (x, y, z), ordered triplet	210
x-coordinate	x = perpendicular distance of P from YZ-plane	210

Formula	Statement	NCERT page
y-coordinate	y = perpendicular distance of P from ZX-plane	210
z-coordinate	z = perpendicular distance of P from XY-plane	210
Origin	O = (0, 0, 0)	210
Point on x-axis	(x, 0, 0)	210
Point on y-axis	(0, y, 0)	210
Point on z-axis	(0, 0, z)	210
Point in XY-plane	(x, y, 0)	210
Point in YZ-plane	(0, y, z)	210
Point in ZX-plane	(x, 0, z)	210
Number of octants	8	209
Octant I sign pattern	(+, +, +)	210
Octant II sign pattern	(−, +, +)	210
Distance formula PQ	$\sqrt{[(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2]}$	212
Distance from origin	$OQ = \sqrt{(x^2 + y^2 + z^2)}$	212
Squared distance	$PQ^2 = (x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2$	211
Collinearity test	PQ + QR = PR ⇒ collinear	212
Pythagoras (right angle at A)	$BC^2 = AB^2 + AC^2$	213
Isosceles test	Exactly two sides equal	213
Locus $PA^2 + PB^2 = 2k^2$	Equation in (x, y, z)	213
Parallelogram check	Opposite sides equal but diagonals unequal	214
Centroid formula	$G = ((x_1+x_2+x_3)/3, (y_1+y_2+y_3)/3, (z_1+z_2+z_3)/3)$	214
Distance is symmetric	$PQ = QP \geqslant 0$	212

2.6 Solved examples (NCERT-grounded)

Example A (NCERT Example 3, p. 212). Find the distance between the points P(1, −3, 4) and Q(−4, 1, 2).

Step 1 — write the coordinate differences: $\Delta x = -4 - 1 = -5$; $\Delta y = 1 - (-3) = 4$; $\Delta z = 2 - 4 = -2$. Step 2 — square and add: $(-5)^2 + 4^2 + (-2)^2 = 25 + 16 + 4 = 45$. Step 3 — take square root: $PQ = \sqrt{45} = 3\sqrt{5}$ units. **Answer:** $3\sqrt{5}$.

Example B (NCERT Example 4, p. 212). Show that the points P(−2, 3, 5), Q(1, 2, 3), R(7, 0, −1) are collinear.

Step 1 — compute PQ: $\sqrt{[(1+2)^2 + (2-3)^2 + (3-5)^2]} = \sqrt{[9+1+4]} = \sqrt{14}$. Step 2 — compute QR and PR: $QR = \sqrt{[(7-1)^2 + (0-2)^2 + (-1-3)^2]} = \sqrt{[36+4+16]} = \sqrt{56} = 2\sqrt{14}$. PR =

$\sqrt{[(7+2)^2 + (0-3)^2 + (-1-5)^2]} = \sqrt{[81+9+36]} = \sqrt{126} = 3\sqrt{14}$. **Step 3 — apply collinearity test:** PQ + QR = $\sqrt{14} + 2\sqrt{14} = 3\sqrt{14}$ = PR, so P, Q, R are collinear. **Answer:** collinear.

Example C (NCERT Example 5, pp. 212–213). Are A(0, 7, 10), B(−1, 6, 6), C(−4, 9, 6) vertices of a right-angled triangle?

Step 1 — compute squared sides: $AB^2 = 1 + 1 + 16 = 18$; $BC^2 = 9 + 9 + 0 = 18$; $CA^2 = 16 + 4 + 16 = 36$. **Step 2 — test Pythagoras:** $AB^2 + BC^2 = 18 + 18 = 36 = CA^2$. ✓ **Step 3 — conclude:** The right angle is at B; △ABC is right-angled and isosceles (AB = BC = $\sqrt{18}$). **Answer:** Yes, right-angled isosceles triangle.

Example D (NCERT Example 6, p. 213). Find the equation of the locus of a point P such that $PA^2 + PB^2 = 2k^2$, where A = (3, 4, 5), B = (−1, 3, −7).

Step 1 — write PA^2 and PB^2: Let P = (x, y, z). $PA^2 = (x-3)^2 + (y-4)^2 + (z-5)^2$. $PB^2 = (x+1)^2 + (y-3)^2 + (z+7)^2$. **Step 2 — expand and add:** $PA^2 + PB^2 = 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + (9+16+25+1+9+49) = 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109$. **Step 3 — set equal to $2k^2$ and simplify:** $2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = 2k^2$, i.e. $x^2 + y^2 + z^2 - 2x - 7y + 2z = k^2 - 109/2$. **Answer:** locus is a sphere (when $k^2 > 109/2$).

Example E (NCERT Example 9, pp. 214–215). The centroid of △ABC is G(1, 1, 1). If A = (3, −5, 7) and B = (−1, 7, −6), find C.

Step 1 — set up centroid formula: ((3 + (−1) + x)/3, (−5 + 7 + y)/3, (7 + (−6) + z)/3) = (1, 1, 1). **Step 2 — solve each coordinate:** (2 + x)/3 = 1 ⇒ x = 1; (2 + y)/3 = 1 ⇒ y = 1; (1 + z)/3 = 1 ⇒ z = 2. **Step 3 — write C:** C = (1, 1, 2). **Answer:** C = (1, 1, 2).

🎯 Practice MCQs

📊 PYQ Alignment

> This chapter is a high-yield CUET (UG) Mathematics topic — typically 4 to 6 MCQs each year, almost all built around the distance formula in 3-D, octant identification using the sign pattern of (x, y, z), and short loci/collinearity problems. Questions tend to be direct numerical applications (often with two given points), recall of which coordinate measures distance from which plane, and statement-based items on properties of axes and coordinate planes.
>
> > **Note on scope:** This Class XI NCERT chapter (Reprint 2026-27) contains **only** the topics above — coordinate axes & planes, octants, coordinates of a point, and the

distance formula. The section formula is no longer part of this chapter in the current NCERT edition, and has therefore not been included.