FREE EDITION · NOTES + 3 SAMPLE MCQS

CUET · MATHEMATICS · CLASS XI · CODE 319

Linear Inequalities

CUET unit: Linear Inequalities

By **UniDrill** · NCERT-grounded study material

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📌 Snapshot

- An inequality is a statement involving $<$, $>$, $\leq$, or $\geq$, unlike an equation which uses $=$ (NCERT §5.1, p. 89).
- Classifies inequalities into strict vs slack, numerical vs literal, and linear (one/two variables) vs quadratic (NCERT §5.2, p. 90).
- Develops algebraic rules for solving linear inequalities in one variable, with the critical rule that the inequality sign reverses on multiplication/division by a negative number (NCERT §5.3, p. 91–92).
- Teaches representation of solution sets on a number line (open circle for strict $<$ / $>$, dark/filled circle for $\leq$ / $\geq$) (Summary, p. 99).
- Applies linear inequalities to word problems — minimum marks for an average, consecutive odd/even integers, acid/boric-acid mixtures, temperature conversion (NCERT §5.3 Examples 7, 8; Misc. Examples 12, 13, p. 94, 97–98).

Note: The Reprint 2026-27 edition of this chapter is confined to linear inequalities in one variable and their graphical representation on the number line; the older sub-sections on linear inequalities in two variables, graphical half-plane solutions, and solution of a system of linear inequalities in two variables are not part of the present PDF (§5.1, p. 89: "we shall confine ourselves to the study of linear inequalities in one and two variables only" — but §5.2 itself, p. 90, also confines the worked treatment to one-variable solutions in the current reprint). MCQs below are therefore grounded only in what the PDF actually contains.

📖 Detailed Notes

2.1 Core concepts

- An **inequality** is formed when two real numbers or two algebraic expressions are related by $<$, $>$, $\leq$, or $\geq$; statements like $30x < 200$ and $40x + 20y \leq 120$ are inequalities (NCERT §5.2, Definition 1, p. 90). They differ from equations in that the solution is typically an interval rather than a single value.
- Inequalities are classified as **numerical** (e.g., $3 < 5$, $7 > 5$) or **literal** (e.g., $x < 5$, $y > 2$); **double inequalities** like $3 < 5 < 7$ or $3 \leq x < 5$ express two relations at once (NCERT §5.2, p. 90).

- Inequalities with $<$ or $>$ are called **strict**; those with $\leq$ or $\geq$ are called **slack** (NCERT §5.2, p. 90). Slack inequalities allow equality; strict ones do not.
- $ax + b < 0$, $ax + b > 0$, $ax + b \leq 0$, $ax + b \geq 0$ (with $a \neq 0$) are **linear inequalities in one variable**; $ax + by < c$, $ax + by > c$, $ax + by \leq c$, $ax + by \geq c$ (with $a \neq 0$, $b \neq 0$) are **linear inequalities in two variables** (NCERT §5.2, p. 90).
- Inequalities involving $ax^2 + bx + c$ are **not linear**; they are quadratic inequalities in one variable (NCERT §5.2, p. 90). Quadratic inequalities have a different solution structure (typically a union of intervals).
- A **solution** of an inequality in one variable is any value of the variable that makes the inequality a true statement; the set of all such values is the **solution set** (NCERT §5.3, p. 91). The solution set may be empty, finite, or an infinite interval.
- **Rule 1:** Equal numbers may be added to (or subtracted from) both sides of an inequality without affecting the sign of inequality (NCERT §5.3, p. 92). This is the basis for transposing terms.
- **Rule 2:** Both sides of an inequality may be multiplied (or divided) by the same **positive** number; but when both sides are multiplied (or divided) by a **negative** number, the sign of inequality is **reversed** (NCERT §5.3, p. 92). Illustration: $3 > 2$, yet $-3 < -2$; $-8 < -7$, yet $(-8)(-2) > (-7)(-2)$, i.e., $16 > 14$ (NCERT §5.3, p. 91). The sign-flip rule is the single most error-prone step.
- Solutions may be sought in **N**, **Z**, or **R**; unless stated otherwise NCERT solves inequalities in **R** (NCERT §5.3, p. 92, after Example 2). The same inequality $30x < 200$ has different solution sets in N (1, 2, ⋯, 6) and R ((–∞, 20/3)).
- Worked algebraic technique: shift like terms, apply Rule 1 then Rule 2, remembering the sign flip; e.g., $5x - 3 < 3x + 1 \Rightarrow 2x < 4 \Rightarrow x < 2$, giving solution set $x \in (-\infty, 2)$ (NCERT §5.3, Example 2, p. 92).
- **Graphical representation on the number line:** for $x < 3$ (from $7x + 3 < 5x + 9$), the solution is shown on the number line in Fig 5.1 (open/hollow circle at 3 and the line darkened to the left of 3) (NCERT §5.3, Example 5, p. 93). For $x \geq 1$ (from $(3x - 4)/2 \geq (x + 1)/4 - 1$), Fig 5.2 shows a dark/filled circle at 1 and the line darkened to the right (NCERT §5.3, Example 6, p. 93–94). The Summary on p. 99 codifies: open circle for $<$ / $>$, dark circle for $\leq$ / $\geq$.
- **Double inequalities** are solved by operating on all three parts simultaneously: $-8 \leq 5x - 3 < 7 \Rightarrow -5 \leq 5x < 10 \Rightarrow -1 \leq x < 2$ (Misc. Example 9, p. 96). The chain notation is shorthand for a system of two inequalities joined by AND.
- **System of inequalities (one variable):** the solution is the intersection of the individual solution sets; e.g., $3x - 7 < 5 + x$ gives $x < 6$, and $11 - 5x \leq 1$ gives $x \geq 2$, so the common solution is $2 \leq x < 6$, shown as the bold portion of the number line in Fig 5.3 (Misc. Example 11, p. 96–97).
- **Word-problem applications:** minimum marks for a target average (Example 7, p. 94: $(62 + 48 + x)/3 \geq 60 \Rightarrow x \geq 70$); consecutive odd natural numbers larger than

10 with sum less than 40 give pairs (11,13), (13,15), (15,17), (17,19) (Example 8, p. 94); a temperature kept between 30°C and 35°C corresponds to 86°F < F < 95°F (Misc. Example 12, p. 97); acid-mixture problem yields `120 < x < 300` litres of 30% acid solution to be added (Misc. Example 13, p. 97–98).

- Two-variable graphical work is outside the current reprint, but its algebra mirrors the one-variable techniques. Two-variable inequalities reappear in Linear Programming (Class XII Ch. 12).

2.2 Definitions to memorise

Term	Definition	Page
Inequality	Two real numbers or expressions related by `<`, `>`, `≤`, `≥`	90
Strict inequality	Uses `<` or `>`	90
Slack inequality	Uses `≤` or `≥`	90
Numerical inequality	Between two numbers, e.g., 3 < 5	90
Literal inequality	Involves a variable, e.g., x < 5	90
Double inequality	Two inequalities joined, e.g., 3 ⩽ x < 5	90
Linear inequality in 1 variable	ax + b ⋚ 0, a ≠ 0	90
Linear inequality in 2 variables	ax + by ⋚ c, a, b ≠ 0	90
Quadratic inequality	Involves $ax^2 + bx + c$, not linear	90
Solution	Value making the inequality true	91
Solution set	Set of all solutions	91
Rule 1	Add/subtract same number; sign preserved	92
Rule 2 (positive)	Multiply/divide by positive number; sign preserved	92
Rule 2 (negative)	Multiply/divide by negative number; sign reversed	92
Open circle convention	Strict `<` / `>` at endpoint	99
Dark circle convention	Slack `≤` / `≥` at endpoint	99
Interval (−∞, a)	x < a	92
Interval (a, ∞)	x > a	92
Interval (−∞, a]	x ⩽ a	92
Interval [a, ∞)	x ⩾ a	92
Interval [a, b]	a ⩽ x ⩽ b	92
Interval (a, b)	a < x < b	92
Domain of solution	N, Z, or R as specified	92

Term	Definition	Page
Intersection of solutions	Solution of a system	96
Empty solution	Inequality with no real solution	90

2.3 Diagrams / processes to remember

- **Fig 5.1 (p. 93):** Number-line graph of $x < 3$ (solution of $7x + 3 < 5x + 9$) — open circle at 3, line darkened to the left.
- **Fig 5.2 (p. 94):** Number-line graph of $x \geq 1$ (solution of $(3x - 4)/2 \geq (x + 1)/4 - 1$) — dark/filled circle at 1, line darkened to the right.
- **Fig 5.3 (p. 97):** Number-line graph of the system $x < 6$ and $x \geq 2$ — bold segment from 2 (dark circle) to 6 (open circle), representing the common solution $2 \leq x < 6$.
- **Procedure to solve a linear inequality in one variable (§5.3, p. 91–92):** (1) transpose like terms using Rule 1, (2) collect the variable on one side, (3) divide by the coefficient using Rule 2, **flipping the sign if the coefficient is negative**, (4) write the solution set in interval form, (5) mark on the number line using open/dark circles per the convention on p. 99.
- **Procedure to solve a system of inequalities:** (1) solve each inequality separately, (2) write each solution set as an interval, (3) intersect the intervals — graphically, take the overlap of the shaded portions on the number line.
- **Procedure to solve a double inequality:** treat as two inequalities joined by AND and operate on all three "parts" simultaneously.
- **Word-problem translation pattern:** define the variable, translate "at least" / "at most" / "less than" / "more than" into $\geqslant$, $\leqslant$, $<$, $>$, set up the inequality, solve, and check the answer makes physical sense (e.g., $x > 0$ for litres).
- **Process — temperature/unit conversion:** substitute the formula into the inequality, multiply/divide each part by the conversion factor, simplify. Direction is preserved because the conversion factor (e.g., 9/5) is positive.

2.4 Common confusions / NTA trap points

- **Forgetting the sign flip** when dividing/multiplying by a negative number — e.g., $-5x \leq -40 \Rightarrow x \geq 8$ (Example 4, p. 93), not $x \leq 8$. NTA distractors often offer the un-flipped option.
- **Open vs dark circle on the number line:** strict $<$ / $>$ uses an open (hollow) circle; slack $\leq$ / $\geq$ uses a filled (dark) circle (Summary, p. 99). Mixing these up is a classic trap.
- **Interval notation:** $x < 2$ corresponds to $(-\infty, 2)$ (open at 2); $x \geq 8$ corresponds to $[8, \infty)$ (closed at 8) (Examples 2 and 4, p. 92–93). Square brackets for slack, parentheses for strict.

- **Domain of the variable matters:** solving $30x < 200$ for natural numbers gives $\{1,2,3,4,5,6\}$, but for integers it also includes 0 and all negative integers (Example 1, p. 92).
- **Solving double inequalities:** every operation must be applied to **all three** members of the chain; e.g., $-8 \le 5x - 3 < 7$ becomes $-5 \le 5x < 10$, not just one side (Misc. Example 9, p. 96).
- **Quadratic inequalities are not linear:** $ax^2 + bx + c \le 0$ is excluded from this chapter (NCERT §5.2, p. 90); a distractor that asks students to classify such an expression as "linear" is a known NTA trap.
- **Forgetting to multiply through to clear fractions.** When the inequality has denominators, multiplying by the LCM of denominators clears them — but check the sign of the multiplier (always positive if denominators are positive constants).
- **Misreading "between A and B" as inclusive.** "Between" in everyday English is ambiguous, but NCERT word problems usually mean strictly between unless "at least"/"at most" is specified.
- **Reversing the inequality when squaring.** Squaring both sides is not a valid operation on inequalities unless both sides are known to be non-negative; the NCERT chapter avoids squaring entirely.
- **Forgetting that an inequality with variables on both sides reduces to a single-side form first.** E.g., $5x - 3 < 3x + 1$ becomes $2x < 4$ before dividing.
- **Treating "or" as intersection.** A system joined by "or" is a union of solution sets, not an intersection.
- **Mis-flipping with negative variables.** If $-x > 0$, then $x < 0$; the flip happens because we multiply by -1.

2.5 Key formulas & theorems

Formula	Statement	NCERT page
Rule 1 (addition)	$a < b \Rightarrow a + c < b + c$	92
Rule 1 (subtraction)	$a < b \Rightarrow a - c < b - c$	92
Rule 2 (positive mult.)	$a < b, c > 0 \Rightarrow ac < bc$	92
Rule 2 (negative mult.)	$a < b, c < 0 \Rightarrow ac > bc$	92
Linear form (1 var)	$ax + b \lesseqgtr 0$	90
Linear form (2 var)	$ax + by \lesseqgtr c$	90
Strict inequalities	$<$ or $>$	90
Slack inequalities	$\leqslant$ or $\geqslant$	90
Open-circle marker	For strict at endpoint	99
Dark-circle marker	For slack at endpoint	99

Formula	Statement	NCERT page
Intersection (system)	Solution of system = ∩ individual solutions	96
Reverse on negation	$x > a \Leftrightarrow -x < -a$	91
Temperature conversion	$C = (5/9)(F - 32)$	97
Mixture concentration	(acid mass)/(total mass) within bounds	97
Average constraint	$(\Sigma \text{ scores})/n \geqslant k$	94
Consecutive odd integers	n, n+2, n+4 with n odd	94
Interval $(-\infty, a)$	$x < a$	92
Interval $[a, \infty)$	$x \geqslant a$	92
Interval (a, b)	$a < x < b$	92
Interval $[a, b]$	$a \leqslant x \leqslant b$	92
Empty solution	Inequality always false (e.g., $x < x$)	90
Universal solution	Inequality always true (e.g., $x < x + 1$)	90
Negation rule	$(a \geqslant b)' = (a < b)$	92
Domain N	Solution restricted to natural numbers	92
Domain Z	Solution restricted to integers	92
Domain R	Solution restricted to reals (default)	92

2.6 Solved examples (NCERT-grounded)

Example A (NCERT Example 2, p. 92). Solve $5x - 3 < 3x + 1$ (real x).

Step 1 — transpose: $5x - 3x < 1 + 3 \Rightarrow 2x < 4$. Step 2 — divide by positive 2: $x < 2$. Step 3 — write in interval form: **$x \in (-\infty, 2)$**.

Example B (NCERT Example 4, p. 93). Solve $(5 - 2x)/3 \leq x/6 - 5$.

Step 1 — multiply by 6 (positive): $2(5 - 2x) \leqslant x - 30 \Rightarrow 10 - 4x \leqslant x - 30$. Step 2 — transpose: $-5x \leqslant -40$. Step 3 — divide by −5 (flip): $x \geqslant 8$. **Answer: $x \in [8, \infty)$**.

Example C (NCERT Misc. Example 9, p. 96). Solve $-8 \leq 5x - 3 < 7$.

Step 1 — add 3 throughout: $-5 \leqslant 5x < 10$. Step 2 — divide by 5 (positive): $-1 \leqslant x < 2$. Step 3 — write: **$x \in [-1, 2)$**.

Example D (NCERT Example 7, p. 94). Find minimum marks Ravi must obtain in the third unit test (62 and 48 in the first two) to maintain an average $\geqslant 60$.

Step 1 — set up: $(62 + 48 + x)/3 \geqslant 60$. Step 2 — multiply by 3: $110 + x \geqslant 180 \Rightarrow x \geqslant 70$. Step 3 — interpret: Ravi must score **at least 70 marks**.

Example E (NCERT Misc. Example 12, p. 97). Convert "temperature between 30°C and 35°C" to °F using $C = (5/9)(F - 32)$.

Step 1 — write: 30 < (5/9)(F − 32) < 35. Step 2 — multiply each part by 9/5: 54 < F − 32 < 63. Step 3 — add 32: **86 < F < 95** °F.

🎯 Practice MCQs

📊 PYQ Alignment

Linear Inequalities is a high-yield CUET (UG) Mathematics topic — CUET 2023, 2024, and 2025 each carried roughly 4–6 MCQs from this chapter, typically focused on (i) solving a one-variable linear inequality (with the negative-coefficient sign-flip trap), (ii) identifying the correct number-line graph (open vs dark circle), (iii) solving a system / double inequality and writing the interval, and (iv) word-problem applications (averages, mixtures, temperature conversion). Direct conceptual questions on the Rules of inequality manipulation also recur.