

CUET · MATHEMATICS · CLASS XI · CODE 319

Permutations and Combinations

CUET unit: Permutations and Combinations

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Snapshot

- Establishes systematic counting techniques: the multiplication (fundamental) principle of counting and factorial notation as the engine for all subsequent formulas.
- Builds permutations (arrangements where order matters) — distinct objects without repetition ($nPr = n!/(n-r)!$), with repetition (n^r), and with some identical objects ($n!/(p_1! p_2! \dots p_k!)$).
- Builds combinations (selections where order does not matter): $nCr = n!/(r!(n-r)!)$, with the bridge identity $nPr = nCr \cdot r!$.
- Derives two key combination identities: $nC(n-r) = nCr$ and Pascal's identity $nCr + nC(r-1) = (n+1)Cr$.
- Heavily applied on CUET in word problems on digits, letter arrangements, committee/team selection, and card-deck selections.

Detailed Notes

2.1 Core concepts

- Counting techniques replace brute-force listing: enumerating all arrangements of 9 remaining digits taken 3 at a time would be tedious, so counting principles avoid enumeration (NCERT §6.1, p. 100). Explicit listing is impractical even for moderate inputs — use multiplication and factorial reasoning instead.
- **Fundamental Principle of Counting (Multiplication Principle):** If an event can occur in m different ways and following it another event can occur in n different ways, then the two events in the given order can occur in $m \times n$ ways; this generalises to any finite number of events (NCERT §6.2, p. 102). Every permutation and combination formula follows from this single principle.
- Illustrative applications: 3 pants \times 2 shirts = 6 pant-shirt pairs; 2 bags \times 3 tiffin boxes \times 2 water bottles = 12 ways to carry items (NCERT §6.2, pp. 100–101). The same logic powers digit-formation, code-generation, and seating problems.
- **Permutation (Definition 1):** an arrangement in a definite order of a number of objects taken some or all at a time (NCERT §6.3, p. 105). The phrase "definite order" is what distinguishes permutations from combinations.
- **Theorem 1 — Permutations of n distinct objects taken r at a time (no repetition):** $nPr = n(n-1)(n-2)\dots(n-r+1)$, proved by filling r vacant places successively

(NCERT §6.3.1, p. 105). The first place can be filled n ways, the second $(n - 1)$ ways, ..., the r -th $(n - r + 1)$ ways; multiply by the fundamental principle.

- **Factorial notation:** $n! = 1 \times 2 \times 3 \times \dots \times (n-1) \times n$; also $n! = n \cdot (n-1)!$. By definition $0! = 1$ (NCERT §6.3.2, p. 105). The recursion $n! = n(n-1)!$ lets one extend the definition backwards and underwrites the convention $0! = 1$.
- **Closed-form for nPr :** $nPr = n!/(n-r)!$, valid for $0 \leq r \leq n$. In particular $nPn = n!/0! = n!$ and $nP0 = 1$ (NCERT §6.3.3, pp. 107–108). The $0! = 1$ convention makes these edge cases consistent.
- **Theorem 2 — Permutations with repetition allowed:** number of permutations of n different objects taken r at a time with repetition is n^r (NCERT §6.3.3, p. 108). Example: 3-letter "words" from NUMBER with repetition = $6^3 = 216$.
- **Theorem 3 (special case):** number of permutations of n objects where p are of the same kind and the rest are all different = $n!/p!$ (NCERT §6.3.4, p. 110). The division by $p!$ removes overcounting from the indistinguishability of the p identical objects.
- **Theorem 4 (general case):** number of permutations of n objects where p_1 are of one kind, p_2 of a second kind, ..., p_k of a k th kind, and the rest are all different = $n!/(p_1! p_2! \dots p_k!)$ (NCERT §6.3.4, p. 110). Worked: ALLAHABAD has 9 letters with 4 A's and 2 L's, giving $9!/(4! 2!) = 7560$ (Example 9, p. 110).
- **Combination:** a selection of objects where order does not matter; for example, from players X, Y, Z only 3 two-player teams exist: XY, YZ, ZX (NCERT §6.4, pp. 114–115). The same 3 teams correspond to 6 ordered pairs ($XY \neq YX$ as permutations) — illustrating $r!$ collapse.
- **Theorem 5 — Relationship between permutations and combinations:** $nPr = nCr \cdot r!$ for $0 < r \leq n$, because each combination of r objects can be rearranged in $r!$ ways (NCERT §6.4, p. 116). This is the conceptual bridge — combinations first, then internally permute.
- **Closed-form for nCr :** $nCr = n!/(r!(n-r)!)$, $0 \leq r \leq n$. By convention $nC0 = 1$ and $nCn = 1$ (NCERT §6.4, p. 116). The symmetry between r and $n - r$ is built into the formula.
- **Identity $nCr = nC(n-r)$:** selecting r objects out of n is the same as rejecting $(n-r)$ objects; also $nCa = nCb \Rightarrow a = b$ or $a + b = n$ (NCERT §6.4, p. 117). This dual rule is heavily tested.
- **Theorem 6 (Pascal's identity):** $nCr + nC(r-1) = (n+1)Cr$ (NCERT §6.4, p. 117, proof on p. 117). Equivalently, the binomial-coefficient Pascal triangle: each entry is the sum of the two above it.
- **Applications worked in chapter:** choosing a Chairman and Vice-Chairman from 12 persons = $12P2 = 132$ (p. 108); 4-card selections from a 52-card deck = $52C4 = 270725$ with breakdowns by suit / colour / face cards (Example 19, pp. 117–119); committee of 3 from 2 men and 3 women = $5C3 = 10$, of which 1-man-2-women =

$2C1 \times 3C2 = 6$ (Example 18, p. 117); team of 5 from 4 girls + 7 boys with at-least-one-of-each = 441 , with at-least-3-girls = 91 (Example 21, p. 120).

- **Recognising the structure** of the problem (arrangement vs selection, repetition allowed or not, identical objects present, restrictions on positions) matters more than memorising formulas.
- Special tally for "no two adjacent" constraints: arrange the non-restricted items first; this creates $n + 1$ "gaps" (including the ends) into which the restricted items can be placed without adjacency. The count is then (gap choices) C (restricted count) \times arrangements of restricted within their chosen gaps.
- A useful sanity check on every count: the answer must be a positive integer. Negative or non-integer results signal a formula error or a forgotten case.
- The "stars and bars" interpretation (not in NCERT but implicit in some Misc. Ex. questions): number of non-negative integer solutions of $x_1 + x_2 + \dots + x_k = n$ equals $(n + k - 1)C(k - 1)$. This generalises the combination idea to "selection with repetition".
- Card-deck counting follows directly from the combination formula nCr : 52 cards split into 4 suits of 13; faces (J, Q, K) total 12 across the deck; choosing r cards from a particular subset uses nCr on that subset's size.

2.7 Worked permutation/combination contrast

NCERT carefully contrasts permutations and combinations in §6.4 (p. 115). Consider the 4 letters A, B, C, D taken 3 at a time. Permutations: $4P3 = 24$ (e.g., ABC, ACB, BAC, BCA, CAB, CBA are six distinct permutations of the same three letters). Combinations: $4C3 = 4$ ({A,B,C}, {A,B,D}, {A,C,D}, {B,C,D}). The ratio $24/4 = 6 = 3!$ confirms the bridge identity $nPr = nCr \cdot r!$. CUET often tests this distinction by inserting the words "arrange" vs "choose" in stems that are otherwise identical.

A second illustration uses number-formation problems. From digits {1, 2, 3, 4, 5}: (i) number of 3-digit numbers without repetition = $5P3 = 60$ (ordered); (ii) number of 3-element subsets of {1, ..., 5} = $5C3 = 10$ (unordered). The ratio again confirms $60 = 10 \cdot 3!$. Whenever a problem refers to "numbers" or "codes" the order matters; when it refers to a "group" or "selection", order does not. Read the stem carefully and underline the key verb to decide which formula applies.

A third illustration: from 6 men and 4 women, choose a committee of 5 with at least 2 women. Approach: split by exactly 2, 3, 4 women.

- Exactly 2 women: $4C2 \cdot 6C3 = 6 \cdot 20 = 120$.
- Exactly 3 women: $4C3 \cdot 6C2 = 4 \cdot 15 = 60$.
- Exactly 4 women: $4C4 \cdot 6C1 = 1 \cdot 6 = 6$. Total = 186. This case-split structure recurs in nearly every "at least" CUET word problem.

2.8 Quick computational shortcuts

- $nC1 = n$; $nC2 = n(n-1)/2$; $nC3 = n(n-1)(n-2)/6$. Memorising these three saves time on many MCQs.
- When n is small (≤ 10) and r is small, expand nCr as the product $(n)(n-1)\cdots(n-r+1)$ divided by $r!$; never compute the full factorials.
- For nCr with $r > n/2$, use symmetry: $nCr = nC(n-r)$. E.g., ${}^{12}C_9 = {}^{12}C_3 = 220$.
- Always check whether the answer-choice list has only one positive integer; combinatorics options are sometimes given with one negative or fractional distractor that is impossible.
- For "letters together" problems, the block count after gluing is $(n - k + 1)!$ where k is the size of the block; then multiply by $k!$ for the internal arrangements.
- For "letters never together" use total – together (complement); for "no two together" use the gaps method.
- For "starts with X and ends with Y" fix the two ends first, then permute the rest in the middle.
- For divisibility constraints on the formed number, fix the last digit (or last two digits) per the divisibility rule first, then count permutations of remaining digits in remaining places. For divisibility by 5, the last digit must be 0 or 5; for divisibility by 2, even; for 4, last two divisible by 4; for 3 or 9, digit sum divisible by 3 or 9 respectively. These number-theoretic constraints cascade neatly into multiplication-principle counts.

2.2 Definitions to memorise

Term	Definition	Page
Fundamental principle of counting	If two events have m and n outcomes, both in succession give $m \times n$	102
Multiplication principle (n events)	Extends to product of all individual counts	102
Factorial $n!$	$1 \cdot 2 \cdot 3 \cdot \cdots \cdot n$	105
Factorial recursion	$n! = n \cdot (n - 1)!$	105
Zero factorial convention	$0! = 1$	105
Permutation	Arrangement in definite order	105
nPr (no repetition)	$n!/(n - r)!$	107
nPr expanded	$n(n - 1)(n - 2) \cdots (n - r + 1)$	105
nPn	$n!$	107
$nP0$	1	107
nPr with repetition	n^r	108
Permutations with p identical	$n!/p!$	110

Term	Definition	Page
Permutations with p_1, p_2, \dots, p_k kinds	$n!/(p_1! p_2! \dots p_k!)$	110
Combination	Selection where order doesn't matter	115
nCr	$n!/(r! (n - r)!)$	116
$nC0$	1	116
nCn	1	116
Bridge identity	$nPr = nCr \cdot r!$	116
Symmetry identity	$nCr = nC(n - r)$	117
Equal-combinations rule	$nCa = nCb \Rightarrow a = b$ or $a + b = n$	117
Pascal's identity	$nCr + nC(r - 1) = (n + 1)Cr$	117
Distinct objects	All n labels different	105
Identical objects	Two or more labels indistinguishable	110
Without replacement	No reuse of selected object	105
With replacement	Same object may be picked again	108

2.3 Diagrams / processes to remember

- **Fig. 6.1 (p. 101):** Tree-style diagram pairing 3 pants P1, P2, P3 with 2 shirts S1, S2 to give 6 outfit combinations — visual proof of the multiplication principle.
- **Fig. 6.2 (p. 101):** Extension to 3 events — 2 bags \times 3 tiffin boxes \times 2 bottles = 12 combinations.
- **ROOT enumeration table (pp. 108–109):** Side-by-side listing showing that treating the two O's as O1, O2 doubles each true arrangement, justifying division by 2! to give $4!/2! = 12$.
- **Fig. 6.3 (p. 115):** Illustration of all three 2-player teams XY, YZ, ZX from {X, Y, Z}, showing that ${}^3C_2 = 3$ (order does not matter).
- **Process — choosing the right formula:** ask three questions. (i) Does order matter? Yes \Rightarrow permutation; No \Rightarrow combination. (ii) Can repetitions occur? If yes, use n^r ; if no, use nPr or nCr . (iii) Are any objects identical? If yes, divide by factorials of repeats.
- **Process — counting word arrangements with restrictions:** treat letters required to stay together as a single block, count arrangements of the reduced set, then multiply by the internal arrangements of the block (Examples 14, 16, pp. 112–113). For "no two X together" problems, place the other letters first and slot the X's into the gaps.

- **Process — selection with constraints:** split into cases by the constraint, compute each case separately, then sum. "At least k" usually means summing from k to maximum, or using total – "less than k".
- **Process — at-least-one and at-most-one:** complement frequently shortens the count. Number with "at least one" = total – number with "none".

2.4 Common confusions / NTA trap points

- Mixing up permutation vs combination — order matters → use nPr ; order does not → use nCr . Word "arrangement"/"sequence"/"queue" signals P; words like "selection"/"committee"/"team"/"choose" signal C.
- Forgetting that $0! = 1$ (definition, p. 105) — leads to wrong values of $nP0$ and $nC0$.
- Treating "with repetition" the same as "without repetition" — repetition allowed gives n^r (Theorem 2, p. 108), not nPr .
- Applying $n!$ directly when some letters repeat — must divide by the factorials of repeated counts: ALLAHABAD = $9!/(4! 2!)$, not $9!$ (Example 9, p. 110).
- Forgetting the digit-0 restriction in number-formation problems — e.g., 3-digit numbers from $\{0,1,2,3,4,5\}$ must exclude those with 0 in the hundreds place (Example 11, p. 110).
- In "vowels together" problems, students often forget to multiply by the internal arrangements of the vowel block (Example 14, p. 112; Example 16(ii), p. 113).
- Confusing $nCa = nCb$: it gives either $a = b$ OR $a + b = n$ (p. 117). Trap: students choose only one branch.
- Mis-counting "at least" by forgetting to subtract overlaps — e.g., "at least one boy AND at least one girl" requires careful complementation, not direct subtraction.
- Mistreating identical objects as distinct — adds spurious factors.
- Forgetting that nCr is symmetric: ${}^{10}C7 = {}^{10}C3$, so compute the smaller- r side for speed.
- Confusing the **position** of repetition: in "no letter is repeated" problems, both repetition and reuse are forbidden, so use nPr .
- Misreading "permutations of all n letters" as $nPn = n!$ when in fact some letters repeat (then it is $n!/p! \dots$).
- Forgetting Pascal's identity is a **binomial-coefficient** identity, not a permutation identity.

2.5 Key formulas & theorems

Formula	Statement	NCERT page
Multiplication principle	$m \times n$ outcomes	102
Factorial	$n! = n(n-1)\dots 1$	105

Formula	Statement	NCERT page
Zero factorial	$0! = 1$	105
Recursion	$n! = n \cdot (n-1)!$	105
nPr	$n!/(n-r)!$	107
nPr (expanded)	$n(n-1)\cdots(n-r+1)$	105
nPn	$n!$	107
$nP0$	1	107
Permutations with repetition	n^r	108
Permutations p identical	$n!/p!$	110
Permutations p_1, \dots, p_r identical	$n!/(p_1! \cdots p_r!)$	110
nCr	$n!/(r!(n-r)!)$	116
$nC0$	1	116
nCn	1	116
Bridge	$nPr = nCr \cdot r!$	116
Symmetry	$nCr = nC(n-r)$	117
Equal-combinations	$nCa = nCb \Rightarrow a = b$ or $a + b = n$	117
Pascal's identity	$nCr + nC(r-1) = (n+1)Cr$	117
ALLAHABAD permutations	$9!/(4!2!) = 7560$	110
Chairman + VC	$12P2 = 132$	108
4-card selection	$52C4 = 270725$	117
ROOT permutations	$4!/2! = 12$	109
Sum of nCr (binomial)	$\sum nCr = 2^n$	117
At-least-one principle	$1 - P(\text{none})$	120
Block method	Glue together, then permute block	112

2.6 Solved examples (NCERT-grounded)

Example A (NCERT Example 1, p. 102). Number of 4-letter words from ROSE without repetition?

Step 1 — 4 distinct letters, 4 places, no repetition: count = $4!$. **Step 2** — evaluate: $4! = 24$. **Step 3** — state: **24 words**.

Example B (NCERT Example 4, p. 103). Number of signals from at least 2 flags out of 5 different flags arranged on a vertical staff?

Step 1 — split by number of flags used: 2, 3, 4, 5. **Step 2** — compute each: $5P2 = 20$; $5P3 = 60$; $5P4 = 120$; $5P5 = 120$. **Step 3** — sum: $20 + 60 + 120 + 120 = \mathbf{320}$.

Example C (NCERT Example 9, p. 110). Permutations of ALLAHABAD?

Step 1 — count letters and repeats: 9 letters total; A repeats 4 times; L repeats 2 times. Step 2 — apply Theorem 4: $9!/(4! \cdot 2!) = 362880/(24 \cdot 2) = 362880/48$. Step 3 — divide: = **7560**.

Example D (NCERT Example 17, p. 117). If ${}^nC_9 = {}^nC_8$, find n .

Step 1 — apply equal-combinations rule: $9 \neq 8$, so use $a + b = n$. Step 2 — compute: $n = 9 + 8 = 17$. Step 3 — state: **$n = 17$** .

Example E (NCERT Example 18, p. 117). Committee of 3 from 2 men and 3 women: number with 1 man and 2 women?

Step 1 — count men selection: ${}^2C_1 = 2$. Step 2 — count women selection: ${}^3C_2 = 3$. Step 3 — multiply: $2 \times 3 = 6$ committees.

Practice MCQs

PYQ Alignment

This chapter is among the highest-yielding CUET (UG) Mathematics units, typically delivering 10–12 MCQs every year. The standard CUET ask is split between (a) direct factorial / nPr / nCr evaluations and small algebraic equations (e.g., find n if $nP_5 = 42 \cdot nP_3$), and (b) word problems on digit-formation, letter-arrangements with repeated letters, committee/team selection with constraints, and card-deck selections — all of which are worked verbatim in §6.3 and §6.4 of this chapter.