FREE EDITION · NOTES + 3 SAMPLE MCQS

CUET · MATHEMATICS · CLASS XI · CODE 319

Probability

CUET unit: Probability

By **UniDrill** · NCERT-grounded study material

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📌 Snapshot

- Builds the axiomatic (Kolmogorov) approach to probability on top of the sample-space/event framework introduced earlier.
- Classifies events (impossible, sure, simple, compound, mutually exclusive, exhaustive) and defines the algebra of events (A', A∪B, A∩B, A–B).
- Establishes the three axioms — $P(A) \geqslant 0$, $P(S)=1$, and additivity for mutually exclusive events — from which $0 \leqslant P(A) \leqslant 1$, $P(\phi)=0$ and $P(A') = 1 - P(A)$ follow.
- Derives the addition theorem $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ and the equally-likely formula $P(E) = n(E)/n(S)$.
- High-yield for CUET because every concept reduces to set operations + arithmetic on counts (dice, cards, coins, committees, arrangements).

📖 Detailed Notes

2.1 Core concepts

- An **event** is any subset E of the sample space S; an event is said to **occur** if the outcome ω of the experiment satisfies $\omega \in E$, otherwise it has not occurred (NCERT §14.1, p. 289–290). Every outcome belongs to S; events are the language for talking about collections of outcomes.
- The empty set ϕ is the **impossible event** (e.g. getting a multiple of 7 on a die) and the whole sample space S is the **sure event** (e.g. getting an odd or even number on a die) (NCERT §14.1.2, p. 290). These are the two extremes; every other event has probability strictly between 0 and 1 (in non-degenerate experiments).
- A **simple (elementary) event** has exactly one sample point; in a sample space of n distinct elements there are exactly n simple events (e.g. tossing two coins yields S = {HH, HT, TH, TT} and four simple events) (NCERT §14.1.2, p. 290).
- A **compound event** has more than one sample point, e.g. in tossing a coin thrice, 'exactly one head appeared' = {HTT, THT, TTH} (NCERT §14.1.2, p. 291).
- **Algebra of events** mirrors set operations: the **complement** $A' = S - A = \{\omega : \omega \in S \text{ and } \omega \notin A\}$ (the event 'not A'); the event **A or B** = $A \cup B$; the event **A and B** = $A \cap B$; the event **A but not B** = $A - B = A \cap B'$ (NCERT §14.1.3, p. 291–292). The link between events and sets is exact, so all set identities (De Morgan, distributivity, ⋯) carry over.

- Two events A and B are **mutually exclusive** if they cannot occur simultaneously, i.e. $A \cap B = \phi$ (A and B are disjoint sets); simple events of any sample space are always mutually exclusive (NCERT §14.1.4, p. 292–293).
- Events $E_1, E_2, \cdots, E_?$ are **exhaustive** if $E_1 \cup E_2 \cup \cdots \cup E_? = S$; if additionally $E_i \cap E_j = \phi$ for all $i \neq j$, they are **mutually exclusive and exhaustive** (NCERT §14.1.5, p. 293). Mutually exclusive AND exhaustive events form a partition of S — the prerequisite for conditional-probability and Bayes' work in Class XII.
- **Axiomatic definition (Kolmogorov):** Probability P is a real-valued function on the power set of S, with range [0, 1], satisfying (i) $P(E) \geqslant 0$ for every event E, (ii) $P(S) = 1$, and (iii) for mutually exclusive events E and F, $P(E \cup F) = P(E) + P(F)$ (NCERT §14.2, p. 295–296). These three axioms are the foundation on which the entire theory rests.
- From axiom (iii) with $F = \phi$ we obtain $\mathbf{P(\phi) = 0}$ (NCERT §14.2, p. 296). This is a theorem, not an axiom — proved by additivity.
- For a finite sample space $S = \{\omega_1, \cdots, \omega_?\}$: $0 \leqslant P(\omega_i) \leqslant 1$, $\Sigma\, P(\omega_i) = 1$, and for any event A, $P(A) = \Sigma\, P(\omega_i)$ for all $\omega_i \in A$; the singleton $\{\omega_i\}$ is the elementary event with probability written $P(\omega_i)$ (NCERT §14.2, p. 296).
- **Equally likely outcomes:** if each of the n simple events has the same probability p, then $np = 1$, so $p = 1/n$; consequently for any event E with $n(E) = m$ favourable outcomes, $\mathbf{P(E) = m/n = n(E)/n(S)}$ (NCERT §14.2.2, p. 299). This is the "classical" definition and is what CUET counts-based problems use.
- **Addition theorem:** for any two events A, B associated with a random experiment, $\mathbf{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$; if A and B are mutually exclusive, $A \cap B = \phi$ so $P(A \cup B) = P(A) + P(B)$, which recovers axiom (iii) (NCERT §14.2.3, p. 299–301).
- **Complement rule:** A and A' are mutually exclusive and exhaustive ($A \cap A' = \phi$, $A \cup A' = S$), so $P(A) + P(A') = 1$, i.e. **P(not A) = 1 – P(A)** (NCERT §14.2.4, p. 301–302).
- Worked illustrations cover one card drawn from 52 (P(diamond) = 1/4, P(not ace) = 12/13, P(black) = 1/2), drawing discs from a bag, and applying De Morgan's law $E' \cap F' = (E \cup F)'$ to find that 'both will not qualify' has probability $1 - P(E \cup F)$ (NCERT §14.2.4 Examples 5–7, p. 302–304).
- Miscellaneous examples extend to hands of cards from a 52-deck using ${}^{52}C_7$, relay-race orderings using permutations, and the three-event addition formula $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(A\cap C) - P(B\cap C) + P(A\cap B\cap C)$ (NCERT Examples 10–12, p. 308–310).
- This topic prepares the ground for Class XII Probability (lemh207), which introduces conditional probability, independence, total probability and Bayes' theorem. Mastery of the Class XI axiomatic framework is essential before moving on.
- A historical note: classical probability (Laplace) and frequency-based probability (von Mises) were the dominant approaches until Kolmogorov's 1933 axiomatisation unified them as a special case of measure theory on the sample space. The Class XI chapter uses the axiomatic framework with finite (or countable) sample spaces.

- The relation between events and sets is exact: every set-theoretic identity (De Morgan, distributive, associative laws) carries over to events. This makes manipulating compound events as easy as manipulating sets.
- Worked counts for the three-coin sample space S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}: |S| = 8; "exactly two heads" = {HHT, HTH, THH} ⇒ P = 3/8; "at least one head" = 1 − P(TTT) = 1 − 1/8 = 7/8. These same patterns underlie countless CUET problems.
- For a single throw of two dice: |S| = 36; "sum = 7" = {(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)} ⇒ P = 6/36 = 1/6. The "diagonal" of constant sum in the 6 × 6 grid gives quick visual counts for any sum.
- For a standard 52-card deck: 4 suits (hearts, diamonds, clubs, spades) × 13 ranks (A, 2, ⋯, 10, J, Q, K). Hearts and diamonds are red (26 cards); clubs and spades are black (26 cards). Face cards: J, Q, K (12 total).
- Outcome counts rely on the multiplication principle from Ch. 6 (Permutations and Combinations): ordered selections without replacement use nPr; unordered selections use nCr; with replacement use n^r. Probability problems are largely counting problems disguised by ratios.
- A useful diagnostic: for equally likely outcomes, probability = (number of favourable outcomes)/(total number of outcomes). If you can count both, you can compute P. The skill lies in correctly identifying what counts as "favourable".
- A subtle remark on infinite sample spaces: the Class XI axiomatic framework strictly applies to finite (or countable) S. Extensions to continuous sample spaces (e.g. picking a real number uniformly from [0, 1]) require measure-theoretic probability, beyond Class XII syllabus.
- Common practical setups: dice (sample space size 6 or 36 for two dice), coins (size 2^n for n tosses), playing cards (52 cards), urn of balls (size = total count), committees (size = nCr possibilities).
- The principle of inclusion-exclusion generalises addition: for n events, the formula has $2^n - 1$ terms, alternating signs. CUET typically restricts to 2 or 3 events, where the formulas are manageable.
- The "atomic" decomposition: in any finite sample space with k atoms (simple events) $\omega_1, \cdots, \omega$⍰, the probability of any event is the sum of the atomic probabilities of its constituent elements. Together with normalisation ($\Sigma\ p_i = 1$), this completely determines the probability measure.

2.2 Definitions to memorise

Term	Definition	Page
Event	Subset E of sample space S	289
Impossible event	ϕ (empty set)	290

Term	Definition	Page
Sure event	S (entire sample space)	290
Simple/elementary event	Singleton subset	290
Compound event	Event with > 1 sample point	291
Complement A'	S – A	291
A or B	$A \cup B$	291
A and B	$A \cap B$	292
A but not B	$A - B = A \cap B'$	292
Mutually exclusive	$A \cap B = \phi$	292
Exhaustive	$E_1 \cup \cdots \cup E_{\boxed{?}} = S$	293
Mutually exclusive & exhaustive	Both conditions: partition	293
Sample space	Set of all outcomes	289
Outcome ω	An element of S	289
Axiom 1	$P(E) \geqslant 0$	296
Axiom 2	$P(S) = 1$	296
Axiom 3	$P(E \cup F) = P(E)+P(F)$ for disjoint	296
$P(\phi)$	0 (derived)	296
Sum-to-1 rule	$\Sigma\ P(\omega_i) = 1$	296
Equally likely formula	$P(E) = n(E)/n(S)$	299
Addition theorem	$P(A \cup B) = P(A)+P(B)-P(A \cap B)$	300
Complement rule	$P(A') = 1 - P(A)$	302
3-event addition	$P(A \cup B \cup C) = \Sigma - \Sigma\text{pair} + P(A \cap B \cap C)$	309
De Morgan (events)	$(A \cup B)' = A' \cap B'$	304
Probability range	$0 \leqslant P(E) \leqslant 1$	296
Validity of assignment	All $p \in [0, 1]$ and $\Sigma = 1$	297

2.3 Diagrams / processes to remember

- **Venn diagram (Fig 14.1, p. 301):** illustrates $A \cup B$ as the disjoint union $(A - B) \cup (A \cap B) \cup (B - A)$, the geometric basis for subtracting $P(A \cap B)$ once to avoid double-counting in the addition theorem.
- **Validity-checklist for probability assignments** (Example 4, p. 297–298): for every assignment verify (i) each $p(\omega_i)$ lies in [0, 1] and (ii) $\Sigma\ p(\omega_i) = 1$; assignments fail if any value is negative, exceeds 1, or the sum $\neq 1$.
- **Coin-toss-thrice sample space** S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT} (p. 291) — the workhorse example for compound events and addition theorem.

- **Two-dice 36-outcome grid** $S = \{(x, y) : x, y = 1, \cdots, 6\}$ (Example 2, p. 294) — used to test mutually exclusive pairs based on sum conditions.
- **Process — compute probability of an event:** (i) list or count S, (ii) list or count favourable outcomes E, (iii) divide. For non-equally-likely outcomes, sum the individual $P(\omega_i)$ for $\omega_i \in E$.
- **Process — compute P(A ∪ B):** (i) check whether $A \cap B = \phi$; (ii) if yes, $P(A) + P(B)$; (iii) if no, $P(A) + P(B) - P(A \cap B)$.
- **Process — compute P(neither A nor B):** apply De Morgan: $P(A' \cap B') = 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)]$.
- **Process — validate a probability assignment:** check each $p_i \in [0, 1]$ AND $\Sigma p_i = 1$. If either fails, the assignment is invalid.

2.4 Common confusions / NTA trap points

- **'Mutually exclusive' vs 'exhaustive'.** Mutually exclusive means pairwise $A \cap B = \phi$; exhaustive means the union equals S. Students collapse these; NTA loves a distractor where events are exhaustive but not disjoint (e.g. A = {1,3,5}, B = {1,2,3}).
- **Dropping the intersection term.** Applying $P(A \cup B) = P(A) + P(B)$ when A and B are NOT mutually exclusive — this overcounts $A \cap B$. Always check whether $A \cap B = \phi$ before using the short form.
- **Confusing 'at least one' with 'exactly one'.** 'At least one will not qualify' = 1 − P(both qualify); 'only one will qualify' = $P(E \cap F') + P(E' \cap F)$ (Example 7, p. 304).
- **De Morgan slip.** $(E \cup F)' = E' \cap F'$, not $E' \cup F'$ — used in computing 'neither E nor F'.
- **Invalid probability assignments.** A value > 1 or < 0, or sum of probabilities ≠ 1, instantly invalidates the assignment (Example 4 and Exercise 14.2 Q1) — NTA frequently disguises this with a single negative entry in a long list.
- **Simple events are always mutually exclusive** (remark, p. 293) — easy to forget when distinguishing simple from compound events.
- **Probability ≠ frequency without replacement.** When drawing without replacement, P changes after each draw; with replacement, P remains constant.
- **Forgetting axiom (ii).** $P(S) = 1$ is the normalisation axiom; an "assignment" satisfying additivity but with $P(S) \neq 1$ is not a probability.
- **Reading "or" as exclusive-or.** In probability, "A or B" is inclusive (union), including the case both occur. Exclusive-or is $(A - B) \cup (B - A)$.
- **Mis-counting card-deck outcomes.** 52 cards = 4 suits × 13 ranks = 12 face cards (J, Q, K) + 4 aces + 36 numbered (2–10). Memorise this breakdown.
- **Forgetting that P(A ∪ B ∪ C) needs inclusion-exclusion.** The three-event formula has six subtractions/additions, not just three.
- **Treating probabilities as counts.** P is a ratio in [0, 1], not a count of outcomes. Confusing "P(E) = 4" with "n(E) = 4" loses marks.

2.5 Key formulas & theorems

Formula	Statement	NCERT page
Axiom 1	$P(E) \geqslant 0$	296
Axiom 2	$P(S) = 1$	296
Axiom 3	$P(A \cup B) = P(A) + P(B)$ if disjoint	296
$P(\phi)$	0	296
Probability bounds	$0 \leqslant P(E) \leqslant 1$	296
Equally likely	$P(E) = m/n$	299
Addition theorem	$P(A\cup B) = P(A)+P(B)-P(A\cap B)$	300
Complement	$P(A') = 1 - P(A)$	302
Three-event inclusion-exclusion	$P(A\cup B\cup C) = \Sigma P - \Sigma$pairwise + P(triple)	309
De Morgan	$(A\cup B)' = A'\cap B'$	304
Disjoint additivity	$P(A) + P(A') = 1$	302
Sum of singleton probabilities	$\Sigma\ P(\omega_i) = 1$	296
Validity check	All $p_i \geqslant 0$ and $\Sigma p_i = 1$	297
Difference event	$A - B = A \cap B'$	292
Symmetric difference	$(A - B) \cup (B - A)$	292
At least one	1 – P(none)	304
Exactly one	$P(A) + P(B) - 2P(A\cap B)$	304
Both	$P(A \cap B)$	292
Probability of complement	$1 - P(A)$	302
52-card draws total	52	302
Suit count	13 per suit	302
Ace count	4	302
Face card count	12	302
Two-coin sample size	4	290
Three-coin sample size	8	291

2.6 Solved examples (NCERT-grounded)

Example A (NCERT Example 5(ii), p. 302). One card drawn from 52. P(not an ace)?

Step 1 — count aces: 4 aces in 52. Step 2 — apply equally-likely: P(ace) = 4/52 = 1/13. Step 3 — apply complement: P(not ace) = 1 − 1/13 = **12/13**.

Example B (NCERT Example 6(v), p. 304). Bag: 4 red, 3 blue, 2 yellow (9 total). P(red or blue)?

Step 1 — mutually exclusive events: red and blue can't co-occur on single draw. Step 2 — sum: P(red) + P(blue) = 4/9 + 3/9. Step 3 — answer: **7/9**.

Example C (Addition theorem worked). P(A) = 0.5, P(B) = 0.7, P(A ∩ B) = 0.3. P(A ∪ B)?

Step 1 — formula: P(A ∪ B) = P(A) + P(B) – P(A ∩ B). Step 2 — substitute: 0.5 + 0.7 – 0.3. Step 3 — compute: **0.9**.

Example D (NCERT Example 7, p. 304). P(E qualifies) = 0.05, P(F qualifies) = 0.10, P(both qualify) = 0.02. P(neither qualifies)?

Step 1 — P(at least one): P(E ∪ F) = 0.05 + 0.10 – 0.02 = 0.13. Step 2 — apply De Morgan: P(E' ∩ F') = 1 – P(E ∪ F). Step 3 — compute: 1 – 0.13 = **0.87**.

Example E (NCERT Example 4(c), p. 297). Is {1/8, 2/3, 1/3, 1/3, −1/4, −1/3} a valid probability assignment?

Step 1 — check non-negativity: two values negative. Step 2 — violates Axiom 1: P(E) ⩾ 0 fails. Step 3 — conclude: **Not valid**.

Example F (NCERT Example 2, p. 294). Two dice thrown; A = "sum is 11", B = "sum is 12". Are A and B mutually exclusive?

Step 1 — list A: {(5,6), (6,5)}. Step 2 — list B: {(6,6)}. Step 3 — intersection: A ∩ B = ϕ ⇒ **mutually exclusive**.

Example G (Coin tossed three times, computational). Find probabilities of (i) exactly one tail (ii) at least one tail.

Step 1 — list S: {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}; |S| = 8. Step 2 — exactly one tail: {HHT, HTH, THH} ⇒ P = 3/8. Step 3 — at least one tail: 1 – P(HHH) = 1 – 1/8 = **7/8**.

🎯 Practice MCQs

📊 PYQ Alignment

> Probability is one of the highest-yield CUET (UG) Mathematics units, with ~8–10 MCQs every year drawn from this chapter and its Class XII counterpart. The most common formats are direct application of P(A∪B) = P(A) + P(B) – P(A∩B), the complement rule,

validity-of-assignment checks (Example 4 / Exercise 14.2 Q1), and equally-likely outcome counting on coins/dice/cards/committee-selection problems.