FREE EDITION · NOTES + 3 SAMPLE MCQS

CUET · MATHEMATICS · CLASS XI · CODE 319

Relations and Functions

CUET unit: Relations and Functions

By **UniDrill** · NCERT-grounded study material

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📌 Snapshot

- Formalises the ordered pair and uses it to build the Cartesian product A × B of two sets, the universe from which relations and functions are carved.
- A relation from A to B is defined as any subset of A × B; a function is a special relation in which every element of the domain has exactly one image in the codomain.
- Catalogues standard real-valued functions with their graphs: identity, constant, polynomial, rational, modulus, signum and greatest integer functions, with their domains and ranges.
- Defines the algebra of real functions (sum, difference, product, quotient and scalar multiple) pointwise on a common domain X, with the quotient requiring g(x) ≠ 0.
- CUET routinely tests: counting elements/subsets/relations from n(A × B) = pq and 2^(pq); identifying which relations are functions; computing domain and range of standard functions.
- Equality of ordered pairs (a, b) = (c, d) ⇔ a = c and b = d is the algebra-friendly handle students miss most often.

📖 Detailed Notes

2.1 Core concepts

Ordered pair. An ordered pair (p, q) groups two elements in a specific order; two ordered pairs are equal iff their first elements are equal and their second elements are equal (NCERT §2.2, p. 25). Thus (2, 3) ≠ (3, 2) although they involve the same numbers, in contrast to sets where order is immaterial.

Cartesian product. For non-empty sets P and Q, P × Q = {(p, q) : p ∈ P, q ∈ Q}; if either P or Q is empty, then P × Q = ϕ (NCERT §2.2, Definition 1, p. 25). Counting rule: if n(A) = p and n(B) = q, then n(A × B) = pq; and if A or B is infinite, A × B is infinite (NCERT §2.2 Remarks (ii)–(iii), p. 26). The triple product A × A × A = {(a, b, c) : a, b, c ∈ A} extends the idea to ordered triplets, and R × R, R × R × R are coordinate systems for the plane and 3-space respectively (NCERT Example 5, p. 27).

Relations. A relation R from a non-empty set A to a non-empty set B is a subset of A × B; the second element of any ordered pair in R is the image of the first element (NCERT

§2.3, Definition 2, p. 28). The set of all first elements is the domain, the set of all second elements is the range, and the entire B is the codomain. Range ⊆ codomain, but they need not be equal (NCERT §2.3, Definitions 3–4, p. 28). Relations can be displayed in roster form, set-builder form, or arrow diagrams.

Counting relations. If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$, and the number of distinct relations from A to B equals the number of subsets of A × B, namely $2^{(pq)}$ (NCERT §2.3 Note, p. 29). For $p = q = 2$, there are $2^4 = 16$ relations.

Functions. A relation f from A to B is a function iff every element of A has one and only one image in B; equivalently, no two distinct ordered pairs of f share the same first element (NCERT §2.4, Definition 5, p. 30). If $(a, b) \in f$, we write $f(a) = b$; b is the image of a and a is the preimage of b. The notation $f : A \to B$ reads "f from A to B". A real-valued function has range in R; a real function has both domain and range in R (Definition 6, p. 31).

Standard real functions. Identity function $f(x) = x$ on R; graph is the 45° line through the origin (Fig 2.8, p. 32). Domain = R, range = R. Constant function $f(x) = c$ on R; graph is a horizontal line at height c (Fig 2.9, p. 32–33). Domain = R, range = {c}. Polynomial function $f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$ where n is a non-negative integer and $a_i \in R$; $x^3 - x^2 + 2$ is polynomial, but $x^{(2/3)} + 2x$ is not (NCERT §2.4.1(iii), p. 33). Graphs of $y = x^2$ (parabola, Fig 2.10) and $y = x^3$ (cubic, Fig 2.11) are standard. Rational function $f(x)/g(x)$ where f, g are polynomial and $g(x) \neq 0$ on the domain. The example $f(x) = 1/x$ defined on R – {0} has both domain and range equal to R – {0} (NCERT Example 15, Fig 2.12, p. 34–35). Modulus function $f(x) = |x|$, equal to x for $x \geqslant 0$ and $-x$ for $x < 0$. Domain = R, range = $[0, \infty)$. V-shaped graph (Fig 2.13, p. 35). Signum function $f(x) = 1$ if $x > 0$, 0 if $x = 0$, -1 if $x < 0$. Domain = R, range = {–1, 0, 1} (Fig 2.14, p. 35–36). Greatest integer function $f(x) = [x]$ is the greatest integer $\leqslant x$. So $[0.999] = 0$, $[1] = 1$, $[-0.5] = -1$. Step graph (Fig 2.15, p. 36).

Algebra of real functions. For real functions f, g defined on a common domain $X \subseteq R$:

- $(f + g)(x) = f(x) + g(x)$
- $(f - g)(x) = f(x) - g(x)$
- $(\alpha f)(x) = \alpha\ f(x),\ \alpha \in R$
- $(fg)(x) = f(x)\ g(x)$
- $(f/g)(x) = f(x)/g(x)$, provided $g(x) \neq 0$ on X (NCERT §2.4.2, p. 36–37).

A linear function $f(x) = mx + c$ is the simplest non-constant polynomial (NCERT Miscellaneous, p. 38). Linear functions appear in every chapter of Class XI/XII mathematics, from coordinate geometry to differential equations.

Why ordered pairs matter. The ordered-pair concept is the only set-theoretic primitive needed to define functions, relations, Cartesian coordinates, and ultimately Euclidean geometry. Kuratowski's set-theoretic definition $(a, b) = \{\{a\}, \{a, b\}\}$ captures order using only set inclusion, but NCERT uses the intuitive notation throughout.

Restrictions and conventions. In all the standard functions, NCERT takes the natural domain (largest set on which the formula makes sense) unless explicitly restricted. For example, the domain of $\sqrt{x}$ is $[0, \infty)$, and the domain of $1/x$ is $R - \{0\}$. Restricting the natural domain produces a new function with possibly different range and properties.

Functions in higher classes. Class XII Relations and Functions (lemh101) builds on this chapter by classifying functions as injective, surjective, and bijective, and develops invertibility. The Class XII chapter on inverse trig functions further restricts domain and codomain of trig functions to achieve invertibility.

Equality of functions. Two functions $f : A \to B$ and $g : C \to D$ are equal iff $A = C$, $B = D$, and $f(x) = g(x)$ for every $x \in A$. Mere equality of formulas is insufficient; domains and codomains must match. Example: $f : R \to R$ with $f(x) = x^2$ and $g : [0, \infty) \to [0, \infty)$ with $g(x) = x^2$ are different functions, although their rules coincide on the smaller domain.

Computation of range — extended technique. Beyond solving $y = f(x)$, the range of a polynomial / rational function can often be read from its graph (parabola opens up vs down) or by completing the square (for quadratics) — $y = ax^2 + bx + c$ has minimum (or maximum) $c - b^2/(4a)$, giving range $[c - b^2/(4a), \infty)$ or $(-\infty, c - b^2/(4a)]$ depending on the sign of a.

2.2 Definitions to memorise

Term	Definition	Page
Ordered pair	(p, q); equal iff first and second elements match	25
Cartesian product $P \times Q$	$\{(p, q) : p \in P, q \in Q\}$; empty if either set is empty	25
$n(A \times B)$	pq, where $p = n(A)$ and $q = n(B)$	26
Ordered triplet	$(a, b, c) \in A \times A \times A$	26
Relation R from A to B	A subset of $A \times B$	28
Domain of R	Set of all first elements of ordered pairs in R	28
Range of R	Set of all second elements of ordered pairs in R	28
Codomain of R	The whole set B; range $\subseteq$ codomain	28
Number of relations A→B	2^(pq) where $p = n(A)$, $q = n(B)$	29
Function $f : A \to B$	Each element of A has exactly one image in B	30
Image / preimage	If $f(a) = b$, b is the image of a; a is the preimage of b	30
Real-valued function	Range is R or a subset of R	31
Real function	Both domain and range are subsets of R	31
Identity function	$f(x) = x$ on R	32
Constant function	$f(x) = c$ on R; range =	32–33
Polynomial function	$f(x) = \Sigma\, a_i x^i$ for non-negative integer powers	33

Term	Definition	Page
Rational function	f(x)/g(x) with f, g polynomial, $g(x) \neq 0$	34
Modulus function	$f(x) = \|x\|$	35
Signum function	f(x) = 1, 0 or −1; range =	35–36
Greatest integer function	f(x) = [x], the largest integer $\leqslant x$	36
Algebra (f/g)(x)	f(x)/g(x) on X provided $g(x) \neq 0$	37
Linear function	f(x) = mx + c, m, c ∈ R	38
Quadratic function	$f(x) = ax^2 + bx + c, a \neq 0$	33
Cubic function	$f(x) = ax^3 + bx^2 + cx + d, a \neq 0$	33
Arrow diagram	Pictorial way to represent a relation/function	28

2.3 Diagrams / processes to remember

NCERT provides a set of standard graph plates that must be memorised exactly — CUET asks you to identify them on sight.

- **Fig 2.1, p. 24** — Arrow-and-pair illustration of A × B for A = {red, blue}, B = {b, c, s}, listing all six ordered pairs.
- **Fig 2.4, p. 28** — Arrow diagram of a relation R from P to Q ("first-letter-of-name" relation).
- **Fig 2.5, p. 29 (Example 7)** — Arrow diagram for R = {(x, y) : y = x + 1} on A = {1, ⋯, 6}; element 6 has no image, hence the relation is not a function on A.
- **Fig 2.8, p. 32** — Graph of identity f(x) = x: 45° line through origin.
- **Fig 2.9, pp. 32–33** — Graph of constant f(x) = c: horizontal line.
- **Fig 2.10, p. 33** — Parabola $y = x^2$; range [0, ∞).
- **Fig 2.11, p. 34** — Cubic $y = x^3$ through origin, symmetric about origin.
- **Fig 2.12, p. 35** — Hyperbola y = 1/x on R − {0}; range R − {0}.
- **Fig 2.13, p. 35** — V-shape $f(x) = |x|$.
- **Fig 2.14, p. 36** — Signum: horizontal segment y = 1 for x > 0, point (0, 0), segment y = −1 for x < 0.
- **Fig 2.15, p. 36** — Step graph of f(x) = [x] with jumps at every integer.
- **Fig 2.16, p. 38** — Linear graph of f(x) = x + 10.
- **Fig 2.17, p. 39** — Piecewise function: 1 − x for x < 0, 1 at 0, 1 + x for x > 0.

Process for finding domain of $\sqrt{f(x)}$. Step 1 — set $f(x) \geqslant 0$. Step 2 — solve the inequality. Step 3 — intersect with the natural domain of f. Example: $\sqrt{(x^2 - 4)} \to x^2 \geqslant 4 \to x \leqslant -2$ or $x \geqslant 2$.

Process for finding range. Method 1: solve y = f(x) for x and find values of y for which x is real. Method 2: trace the graph mentally.

2.4 Common confusions / NTA trap points

- **A × B vs B × A.** Same cardinality but different sets — ordered pairs are order-sensitive (NCERT §2.2 Example 2, p. 26).
- **Number of relations.** It is 2^(n(A)·n(B)), not 2^(n(A)+n(B)) (NCERT §2.3 Note, p. 29).
- **Relation vs function.** A relation fails to be a function if any first element has no image OR more than one image (NCERT Examples 7, 8, 11).
- **Range vs codomain.** Range ⊆ codomain; they need not be equal.
- **Signum range.** Exactly {−1, 0, 1}, NOT [−1, 1].
- **Greatest integer subtlety.** [x] = n for n ⩽ x < n+1 (half-open). So [0.999] = 0, not 1; [−0.5] = −1, not 0.
- **Quotient domain.** For f/g, exclude all x where g(x) = 0, even outside the natural domain of f.
- **Modulus range.** [0, ∞), not R.
- **Polynomial restriction.** Powers must be non-negative integers — x^(2/3) is NOT a polynomial.
- **Equal functions.** Two functions are equal iff they have the same domain AND f(x) = g(x) for all x in that domain.
- **f(a + b) ≠ f(a) + f(b) in general** — assuming this is the most common algebraic blunder.
- **$f^2(x)$ means f(x)·f(x), not f(f(x))** in NCERT notation unless context clarifies.

2.5 Key formulas & theorems

Formula / Theorem	Statement	NCERT page
Equality of ordered pairs	(a, b) = (c, d) ⇔ a = c, b = d	25
Cardinality of A × B	n(A × B) = n(A)·n(B)	26
Cardinality of relations	2^(n(A)·n(B))	29
Identity function	f(x) = x; domain = range = R	32
Constant function	f(x) = c; range =	32
Modulus function	|x| = x if x ⩾ 0, −x if x < 0	35
Signum function	sgn(x) = 1, 0, −1 per sign	35
Greatest integer	[x] = n if n ⩽ x < n+1	36
(f + g)(x)	f(x) + g(x)	36
(f − g)(x)	f(x) − g(x)	36

Formula / Theorem	Statement	NCERT page
$(\alpha f)(x)$	$\alpha\ f(x)$	36
$(f \cdot g)(x)$	$f(x) \cdot g(x)$	37
$(f/g)(x)$	$f(x)/g(x)$, $g(x) \neq 0$	37
Domain of $1/x$	R –	34
Range of $1/x$	R –	34
Domain of $\sqrt{x}$	$[0, \infty)$	(impl.)
Range of x^2	$[0, \infty)$	33
Range of $\|x\|$	$[0, \infty)$	35
Range of $[x]$	Z	36
Linear function	$f(x) = mx + c$	38
$n(A \times A \times A)$	$n(A)^3$	26
Distributive of × over ∪	$A \times (B \cup C) = (A \times B) \cup (A \times C)$	(Misc)
$n(R \times R)$	uncountable infinite (the plane)	27

2.6 Solved examples

Example 1 — Ordered-pair equation. If $(x + 1, y - 2) = (3, 1)$, find x and y. Step 1 — Equate first components: $x + 1 = 3 \Rightarrow x = 2$. Step 2 — Equate second components: $y - 2 = 1 \Rightarrow y = 3$. **Answer:** $x = 2$, $y = 3$.

Example 2 — Counting relations. If $A = \{1, 2, 3\}$ and $B = \{a, b\}$, how many relations are there from A to B? Step 1 — $n(A \times B) = 3 \times 2 = 6$. Step 2 — Number of relations = $2^6 = 64$. **Answer:** 64.

Example 3 — Domain of a rational function. Find the domain of $f(x) = (x^2 + 1)/(x^2 - 3x + 2)$. Step 1 — Denominator = $x^2 - 3x + 2 = (x - 1)(x - 2)$. Step 2 — Denominator zero at $x = 1, 2$. Step 3 — Domain = $R - \{1, 2\}$. **Answer:** $R - \{1, 2\}$.

Example 4 — Algebra of functions. If $f(x) = x^2$ and $g(x) = 2x + 1$, compute $(f - g)(x)$ and $(f/g)(x)$. Step 1 — $(f - g)(x) = x^2 - (2x + 1) = x^2 - 2x - 1$. Step 2 — $(f/g)(x) = x^2/(2x + 1)$, $x \neq -1/2$. **Answer:** $x^2 - 2x - 1$ and $x^2/(2x + 1)$ with $x \neq -1/2$.

Example 5 — Greatest integer. Evaluate $[3.7] + [-2.3] + [0]$. Step 1 — $[3.7] = 3$. Step 2 — $[-2.3] = -3$ (since $-3 \leqslant -2.3 < -2$). Step 3 — $[0] = 0$. Sum = $3 - 3 + 0 = 0$. **Answer:** 0.

🎯 Practice MCQs

📊 PYQ Alignment

A perennial CUET (UG) Mathematics favourite contributing roughly 6–8 MCQs across 2023–2025 papers. Recurring question types: counting n(A × B), counting relations or subsets, identifying which relation is a function, stating the domain or range of standard functions — especially modulus, signum, greatest integer and rational functions defined by 1/x or 1/(polynomial). Assertion-reason items on greatest-integer evaluation are routine.

For the full PYQ archive across all Mathematics chapters, see **/pyq/mathematics**.