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CUET · MATHEMATICS · CLASS XI · CODE 319

Sequences and Series

CUET unit: Sequences and Series

By **UniDrill** · NCERT-grounded study material

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📌 Snapshot

- A sequence is an ordered list (a function on the natural numbers) and a series is the sum of its terms, written compactly using sigma notation (NCERT §8.2–§8.3).
- A **Geometric Progression** has a constant ratio, general term $a_n = ar^{(n-1)}$, and sum $S_n = a(r^n - 1)/(r - 1)$ for $r \neq 1$ (NCERT §8.4.1–§8.4.2).
- The **Geometric Mean** of two positive numbers is $\sqrt{ab}$; to insert n GMs between two positive numbers use $r = (b/a)^{(1/(n+1))}$ (NCERT §8.4.3).
- It proves the inequality **A.M. ⩾ G.M.** for two positive numbers using $(\sqrt{a} - \sqrt{b})^2 \geq 0$ (NCERT §8.5).
- CUET regularly tests direct application: finding n th term/sum of a GP, inserting GMs, and using the AM–GM relationship to recover the two numbers.

📖 Detailed Notes

2.1 Core concepts

- A **sequence** is an ordered list of numbers $a_1, a_2, a_3, \ldots, a_n, \ldots$; subscripts denote position and a_n is called the **general term** or n th term (NCERT §8.2, p. 137). The word "sequence" emphasises order; rearranging the terms generally produces a different sequence.
- A sequence with a finite number of terms is a **finite sequence**; otherwise it is an **infinite sequence**. Example: ancestors over 10 generations 2, 4, 8, ⋯, 1024 is finite; the successive quotients 3, 3.3, 3.33, ⋯ in dividing 10 by 3 form an infinite sequence (NCERT §8.2, p. 136–137).
- Terms of a sequence need not follow a closed formula. Sometimes they are defined by a **recurrence relation**, e.g. the Fibonacci sequence $a_1 = a_2 = 1$, $a_n = a_{(n-1)} + a_{(n-2)}$ for $n > 2$; primes 2, 3, 5, 7, ⋯ have no formula at all (NCERT §8.2, p. 136–137). Both are valid sequences's sense.
- A sequence can be regarded as a **function** whose domain is the set of natural numbers (or a subset of it); sometimes $a(n)$ is used in place of a_n (NCERT §8.2, p. 137). This functional view is what makes sequences amenable to calculus (limits, convergence) in Class XII and beyond.

- A **series** associated with a sequence $a_1, a_2, \ldots, a_n, \ldots$ is the expression $a_1 + a_2 + \ldots + a_n + \ldots$. It is finite or infinite according to the parent sequence, and is written compactly as $\Sigma_{(k=1)}^n a_k$ using the Greek letter sigma (NCERT §8.3, p. 137).
- The word series refers to the indicated sum, not to the sum itself; for example $1 + 3 + 5 + 7$ is a four-term series whose sum is 16 (NCERT §8.3, Remark, p. 137). CUET sometimes asks "write the series" — meaning write the indicated sum, not just the value.
- A sequence $a_1, a_2, \ldots$ is a **Geometric Progression (G.P.)** if each term is non-zero and $a_{(k+1)}/a_k = r$ (constant) for $k \geq 1$; a is the first term and r the common ratio, so the GP is $a, ar, ar^2, ar^3, \ldots$ (NCERT §8.4, p. 139). The common ratio can be positive, negative, or a non-zero fraction, but never 0 (else the sequence terminates).
- **General term of a GP:** $a_n = ar^{(n-1)}$. A finite GP is $a, ar, \ldots, ar^{(n-1)}$; an infinite GP is $a, ar, ar^2, \ldots, ar^{(n-1)}, \ldots$ (NCERT §8.4.1, p. 140). The exponent of r is $n - 1$, not n — the single most common student error.
- **Sum to n terms of a GP:** If $r = 1$, $S_n = na$. If $r \neq 1$, $S_n = a(r^n - 1)/(r - 1)$, equivalently $a(1 - r^n)/(1 - r)$, derived by subtracting rS_n from S_n (NCERT §8.4.2, p. 140). The "easier" form depends on whether $|r| > 1$ or $|r| < 1$; both are algebraically identical.
- Standard worked examples that CUET mirrors: ancestors over 10 generations $2 + 4 + 8 + \cdots + 2^{10} = 2046$ (Example 11, p. 143); bacteria doubling every hour; quarterly bank interest. The pattern is always to identify a, r, n and plug into the sum formula.
- **Geometric Mean (G.M.)** of two positive numbers a and b is $\sqrt{(ab)}$. Three numbers a, G, b are in GP iff $G = \sqrt{(ab)}$; e.g. G.M. of 2 and 8 is 4, and 2, 4, 8 is a GP (NCERT §8.4.3, p. 143). G.M. is undefined (or complex) if either of a, b is negative.
- **Inserting n GMs between a and b:** If $G_1, G_2, \ldots, G_n$ lie between positive numbers a and b so that $a, G_1, G_2, \ldots, G_n, b$ is a GP, then b is the $(n+2)$th term: $b = ar^{(n+1)}$, giving $r = (b/a)^{(1/(n+1))}$; the GMs are $G_k = a(b/a)^{(k/(n+1))}$ (NCERT §8.4.3, p. 143). The total number of terms is n + 2 because a and b are also part of the GP.
- **Relationship A.M. ⩾ G.M.:** For two positive numbers a, b, let $A = (a+b)/2$ and $G = \sqrt{(ab)}$. Then $A - G = (a + b - 2\sqrt{(ab)})/2 = (\sqrt{a} - \sqrt{b})^2/2 \geq 0$, hence $A \geq G$ (NCERT §8.5, p. 144). Equality holds iff $\sqrt{a} = \sqrt{b}$, i.e. $a = b$. The inequality generalises to n positive numbers (n-variable AM-GM), but the NCERT chapter restricts to two.
- Arithmetic-progression material (covered in Class X) and the convergence of infinite GPs are out of scope here (sum-to-infinity $S_\infty = a/(1 - r)$ for $|r| < 1$ is in the older syllabus and may appear as Misc. Exercise but is not central in the 2026-27 edition).

2.2 Definitions to memorise

Term	Definition	Page
Sequence	Ordered arrangement of numbers $a_1, a_2, \ldots$; a function on N	136–137
General (nth) term	The number a_n at the nth position	136
Finite sequence	Sequence with finitely many terms	137
Infinite sequence	Sequence that is not finite	137
Fibonacci sequence	$a_1 = a_2 = 1; a_n = a_{(n-1)} + a_{(n-2)}$ for n > 2	136
Recurrence relation	Equation defining a_n from earlier terms	136
Series	Sum $a_1 + a_2 + \ldots + a_n + \ldots$	137
Sigma notation	$\Sigma_{(k=1)}^n a_k = a_1 + a_2 + \ldots + a_n$	137
Finite series	Series associated with a finite sequence	137
Infinite series	Series associated with an infinite sequence	137
G.P.	Non-zero sequence with constant ratio $r = a_{(k+1)}/a_k$	139
First term a of GP	The number a_1	139
Common ratio r	Constant ratio between consecutive terms	139
General term of GP	$a_n = ar^{(n-1)}$	140
Finite GP	$a, ar, \ldots, ar^{(n-1)}$	140
Infinite GP	$a, ar, ar^2, \ldots$	140
Sum of n terms (r ≠ 1)	$S_n = a(r^n - 1)/(r - 1)$	140
Sum of n terms (r = 1)	$S_n = na$	140
Geometric Mean (G.M.)	$G = \sqrt{(ab)}$ for $a, b > 0$	143
Three terms in GP	a, G, b in GP iff $G^2 = ab$	143
Common ratio inserting n GMs	$r = (b/a)^{(1/(n+1))}$	143
Insert n GMs (k-th)	$G_k = a (b/a)^{(k/(n+1))}$	143
Arithmetic Mean	$A = (a + b)/2$	144
A.M.–G.M. inequality	$(a + b)/2 \geq \sqrt{(ab)}$ for $a, b > 0$	144
Equality condition	A.M. = G.M. iff a = b	144
Sigma identity	Sum compactly written with Σ; index begins at 1 unless stated	137

2.3 Diagrams / processes to remember

- Portrait of **Fibonacci (1175–1250)** appears alongside the introduction (NCERT §8.1, p. 135). Sequences and series model real-world growth and decay despite their abstract appearance.
- **Two-step derivation of the GP sum (NCERT §8.4.2, p. 140):** (i) write $S_n = a + ar + \ldots + ar^{n-1}$; (ii) multiply by r to get $rS_n = ar + ar^2 + \ldots + ar^n$; subtract the first from the second to obtain $(r - 1)S_n = a(r^n - 1)$. Memorise the derivation: CUET sometimes asks for a sketch of it as a statement-correctness MCQ.
- The algebraic identity $A - G = (\sqrt{a} - \sqrt{b})^2/2$, which makes the AM–GM inequality visible at one glance (NCERT §8.5, p. 144). The square is non-negative ⇒ $A - G \geqslant 0$ ⇒ $A \geqslant G$.
- **Worked Example 11 (ancestors):** $a = 2, r = 2, n = 10$, so $S_{10} = 2(2^{10} - 1) = 2046$, illustrating a real-life GP sum (NCERT §8.4.2, p. 143). This is the prototype for every "doubling over n periods" question.
- **Process — identify GP, then plug:** (i) Read first term a and common ratio r from the sequence. (ii) Decide whether the question asks for a_n (use general term) or S_n (use sum formula). (iii) Substitute and simplify; check whether $r = 1$ to avoid division by zero.
- **Process — insert n GMs:** (i) Treat a and b as the first and $(n+2)$-th terms of a GP with n+1 ratios in between. (ii) Compute $r = (b/a)^{1/(n+1)}$. (iii) The k-th GM is $a \cdot r^k$.
- **Process — find two numbers from A.M., G.M.:** Given A and G, use $a + b = 2A$ and $ab = G^2$. Then a, b are roots of $t^2 - 2A\,t + G^2 = 0$. Discriminant $4A^2 - 4G^2 \geq 0$ is guaranteed by AM-GM.

2.4 Common confusions / NTA trap points

- **Sigma notation vs. value.** A series is the indicated sum, not the number that results. $1 + 3 + 5 + 7$ is the series; 16 is its sum (NCERT §8.3 Remark, p. 137). NTA exploits this by asking what the "series" is.
- n **th term off-by-one.** In a GP, $a_n = ar^{n-1}$, not ar^n. Students who write $a_{10} = ar^{10}$ lose marks on direct questions (NCERT §8.4.1, p. 140).
- **Wrong sum formula when** $r = 1$**.** $S_n = a(r^n - 1)/(r - 1)$ is undefined when $r = 1$; in that case $S_n = na$ (NCERT §8.4.2, p. 140). A common trap option is $n \cdot a$ paired with the $r \neq 1$ formula in the same MCQ.
- **G.M. sign.** G.M. of two positive numbers is the positive square root $\sqrt{ab}$ (NCERT §8.4.3, p. 143). When inserting GMs the common ratio may admit two real values (e.g. $r = \pm 4$ between 1 and 256), but the definition of GM picks the positive root.
- **A.M. ⩾ G.M. holds only for positive numbers.** The proof relies on $(\sqrt{a} - \sqrt{b})^2$, which needs $a, b > 0$ (NCERT §8.5, p. 144). For mixed-sign numbers the inequality may reverse or be undefined.

- **GP requires non-zero terms.** A sequence with a zero term is excluded from being a GP because the ratio `a_(k+1)/a_k` would be undefined (NCERT §8.4, p. 139).
- **Confusing AP and GP general terms.** AP: `a_n = a + (n − 1)d`. GP: `a_n = a · r^(n−1)`. The structural difference is addition vs. multiplication.
- **Mis-counting inserted means.** Inserting n GMs between a and b gives a GP with `n + 2` terms total. Many students treat the count as n + 1.
- **Mistaking G.M. for the average of GMs.** G.M. of a and b is `√(ab)`, not `(√a + √b)/2`.
- **Mis-applying sum formula to infinite GP.** The finite-sum formula `S_n` is unchanged; the limit as n → ∞ requires `|r| < 1` and is `a/(1 − r)`.
- **Forgetting domain restrictions for n.** n is a positive integer; sums like `S_0` or `S_{-1}` are not defined.
- **Misreading common ratio.** In 8, 4, 2, 1, ⋯, `r = 1/2`, not 2. Read direction carefully.
- **Confusing exponent base.** `a · 4^(n−1) = …` and `a · 2^(n−1) = …` produce different n's; identify r correctly before solving.
- **Treating G.M. as signed.** Even if a, b satisfy `ab > 0` while both being negative, G.M. is defined in NCERT only for positive a, b.
- **Off-by-one in inserting means.** Common-ratio formula `r = (b/a)^(1/(n+1))` has denominator `n + 1`, not `n` — corresponding to the n + 1 gaps between n + 2 terms.
- **Equating sum of GP to product.** `S_n` is a sum, not a product of terms; the product of n GP terms is `(a · ar^(n−1))^(n/2)` (geometric structure, not in the syllabus but a common distractor).

2.5 Key formulas & theorems

Formula	Statement	NCERT page
Sequence	Function from N to R, written `a_n`	137
Series	`a_1 + a_2 + … + a_n + …`	137
Sigma notation	`Σ_(k=1)^n a_k`	137
Fibonacci recurrence	`a_n = a_(n−1) + a_(n−2)`, `a_1 = a_2 = 1`	136
GP definition	`a_(k+1)/a_k = r` (constant)	139
nth term of GP	`a_n = a r^(n−1)`	140
Sum of n GP terms (r ≠ 1)	`S_n = a(r^n − 1)/(r − 1)`	140
Sum of n GP terms (r = 1)	`S_n = n a`	140
Alternate sum form	`S_n = a(1 − r^n)/(1 − r)`	140
Geometric mean	`G = √(ab)`	143

Formula	Statement	NCERT page
3 terms in GP test	$G^2 = ab$	143
Common ratio for n GMs	`r = (b/a)^(1/(n+1))`	143
k-th inserted GM	`G_k = a(b/a)^(k/(n+1))`	143
Arithmetic mean	`A = (a + b)/2`	144
AM ⩾ GM	`(a + b)/2 ≥ √(ab)`	144
Equality condition	A = G iff a = b	144
A − G formula	$A - G = (\sqrt{a} - \sqrt{b})^2/2$	144
Two numbers from A, G	Roots of $t^2 - 2At + G^2 = 0$	144
Sum of ancestors GP	$S_{10} = 2(2^{10} - 1) = 2046$	143
GP product property	If a, b, c in GP, then $b^2 = ac$	139
Common ratio range	r ≠ 0 (else GP terminates)	139
GP doubling	a = 1, r = 2 ⇒ `a_n = 2^(n-1)`	140
GP halving	r = 1/2 ⇒ `a_n = a/2^(n-1)`	140
Number of terms inserted	a, G_1, ⋯, G_n, b ⇒ n + 2 total	143
Sigma over constant	`Σ_(k=1)^n c = n c`	137

2.6 Solved examples (NCERT-grounded)

Example A (NCERT Example 4, p. 140). Find the 10th term of the GP 5, 25, 125, ⋯

Step 1 — identify a, r: a = 5, r = 25/5 = 5. Step 2 — apply general term: $a_{10} = a \cdot r^9 = 5 \cdot 5^9 = 5^{10}$. Step 3 — answer: $\mathbf{5^{10}}$ (≈ 9 765 625).

Example B (NCERT Example 5, p. 141). Which term of the GP 2, 8, 32, ⋯ equals 131072?

Step 1 — identify a, r: a = 2, r = 8/2 = 4. Step 2 — set up equation: $2 \cdot 4^{(n-1)} = 131072 \Rightarrow 4^{(n-1)} = 65536$. Step 3 — solve: $4^8 = 65536$, so n − 1 = 8 ⇒ n = 9. **Answer:** 9th term.

Example C (NCERT Example 11, p. 143). Find the sum of ancestors over 10 generations (parents, grandparents, ⋯).

Step 1 — model: GP with a = 2 (parents), r = 2 (each generation doubles), n = 10. Step 2 — apply sum formula: $S_{10} = 2(2^{10} - 1)/(2 - 1) = 2(1024 - 1) = 2 \cdot 1023$. Step 3 — answer: S_{10} = **2046** ancestors.

Example D (NCERT Example 12, p. 144). Insert three numbers between 1 and 256 to form a GP with positive ratio.

Step 1 — model: 1, G_1, G_2, G_3, 256 is a 5-term GP, so $256 = 1 \cdot r^4$. Step 2 — solve for r: $r^4 = 256 \Rightarrow r = 4$ (positive root). Step 3 — compute means: $G_1 = 4$, $G_2 = 16$, $G_3 = 64$. **Answer:** 4, 16, 64.

Example E (NCERT Example 13, p. 144–145). Find two positive numbers whose A.M. is 10 and G.M. is 8.

Step 1 — write system: $a + b = 20$; $ab = 64$. Step 2 — form quadratic: a, b are roots of $t^2 - 20t + 64 = 0$; discriminant = 400 − 256 = 144. Step 3 — solve: $t = (20 \pm 12)/2 = 16$ or 4. **Answer:** 4 and 16.

🎯 Practice MCQs

📊 PYQ Alignment

> Sequences and Series is among the highest-yield CUET (UG) Mathematics units: roughly 8–10 MCQs each year (2023–25), with direct questions on the nth term and sum of a GP (Examples 4–7), insertion of geometric means (Example 12), and recovery of two numbers from their A.M. and G.M. (Example 13). Expect at least one assertion–reason or statement-based question pinned on the A.M. ⩾ G.M. inequality and one application item modelled on the "ancestors" or "bacteria-doubling" GP sums in §8.4.2.