FREE EDITION · NOTES + 3 SAMPLE MCQS

CUET · MATHEMATICS · CLASS XI · CODE 319

Straight Lines

CUET unit: Straight Lines

By **UniDrill** · NCERT-grounded study material

WWW.UNIDRILL.IN

📌 Snapshot

- Extends the coordinate geometry of earlier classes (distance, section formula, area of triangle) into the algebraic study of the simplest curve — the straight line.
- Builds the central idea of **slope** ($m = \tan\theta$) as the algebraic measure of a line's inclination and uses it to characterise parallelism, perpendicularity and the angle between two lines.
- Develops five standard forms of a line's equation (point-slope, two-point, slope-intercept, intercept, and the general form $Ax + By + C = 0$) and the perpendicular-distance formulas.
- A reliable CUET scoring zone: questions reduce to plug-and-chug on slope, angle, distance and equation-form conversions — formula-heavy but logic-light.

📖 Detailed Notes

2.1 Core concepts

- **Recap formulae carried into this chapter:** distance $PQ = \sqrt{[(x_2-x_1)^2 + (y_2-y_1)^2]}$; internal section formula $((mx_2+nx_1)/(m+n), (my_2+ny_1)/(m+n))$; midpoint $((x_1+x_2)/2, (y_1+y_2)/2)$; area of triangle $= \frac{1}{2} |x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)|$; three points are collinear iff this area is zero (NCERT §9.1, pp. 151–152). These are the building blocks for every straight-line formula that follows.
- **Inclination and slope:** the angle θ measured anticlockwise from the positive x-axis to a line is its inclination, with $0° \leqslant \theta \leqslant 180°$. The slope is $m = \tan\theta$ ($\theta \neq 90°$). Lines parallel to the x-axis have inclination 0° (slope 0); vertical lines have inclination 90° and undefined slope (NCERT §9.2, pp. 152–153). The slope is a real number that captures "rise over run".
- **Slope from two points:** for points (x_1, y_1) and (x_2, y_2) on a non-vertical line, $m = (y_2 - y_1)/(x_2 - x_1)$, valid for both acute and obtuse inclinations (NCERT §9.2.1, pp. 153–154). Order of points does not matter because both numerator and denominator change sign simultaneously.
- **Parallelism and perpendicularity:** two non-vertical lines are parallel iff $m_1 = m_2$; they are perpendicular iff $m_1 m_2 = -1$, i.e. their slopes are negative reciprocals (NCERT §9.2.2, pp. 154–155). These two tests cover almost every "find the line through P parallel/perpendicular to L" CUET problem.

- **Angle between two lines:** the acute angle θ between non-vertical lines with slopes m_1 and m_2 satisfies $\tan\theta = |(m_2 - m_1)/(1 + m_1m_2)|$, provided $1 + m_1m_2 \neq 0$. The obtuse angle is $\phi = 180^\circ - \theta$ (NCERT §9.2.3, pp. 156–157). When $1 + m_1m_2 = 0$, the lines are perpendicular and $\theta = 90^\circ$.
- **Collinearity via slopes:** three points A, B, C are collinear iff slope of AB = slope of BC (NCERT Summary, p. 174 and §9.2 Example/Exercise discussion). Equivalently, the area of triangle ABC is 0.
- **Horizontal and vertical lines:** a horizontal line at distance a from the x-axis has equation y = a or y = –a; a vertical line at distance b from the y-axis has equation x = b or x = –b (NCERT §9.3.1, pp. 159–160). These are the two cases that "elude" the general slope-intercept form.
- **Point-slope form:** the line through (x_0, y_0) with slope m is $y - y_0 = m(x - x_0)$ (NCERT §9.3.2, pp. 160–161). Use when a point and slope are known directly.
- **Two-point form:** the line through (x_1, y_1) and (x_2, y_2) is $y - y_1 = [(y_2 - y_1)/(x_2 - x_1)]\,(x - x_1)$ (NCERT §9.3.3, p. 161). Use when two points are known but slope is not separately given.
- **Slope-intercept form:** with slope m and y-intercept c, the line is y = mx + c; with x-intercept d, the line is y = m(x – d) (NCERT §9.3.4, pp. 161–162). This is the form in which graphs of linear functions are most naturally read.
- **Intercept form:** a line with x-intercept a and y-intercept b is x/a + y/b = 1 (NCERT §9.3.5, p. 163). a and b are signed — negative intercepts produce negative a or b.
- **General form Ax + By + C = 0:** any equation of this form (A, B not simultaneously zero) represents a line; its slope is –A/B (when B ≠ 0); used to derive standard forms by reduction (NCERT §9.3.5, p. 163 and §9.4 application). Every line in the plane can be expressed in this form.
- **Distance of a point from a line:** the perpendicular distance d from $P(x_1, y_1)$ to the line Ax + By + C = 0 is $d = |Ax_1 + By_1 + C|/\sqrt{A^2 + B^2}$ (NCERT §9.4, pp. 164–166). The modulus is essential; distance is non-negative.
- **Distance between two parallel lines:** for $y = mx + c_1$ and $y = mx + c_2$, $d = |c_1 - c_2|/\sqrt{1 + m^2}$; for $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$, $d = |C_1 - C_2|/\sqrt{A^2 + B^2}$ (NCERT §9.4.1, pp. 166–167). The A, B coefficients must match exactly before applying.
- **Reductions between forms:** Ax + By + C = 0 converts to slope-intercept y = –(A/B)x – C/B, to intercept form by dividing by –C: x/(–C/A) + y/(–C/B) = 1, and so on. CUET often tests these conversions directly.

2.2 Definitions to memorise

Term	Definition	Page
Inclination	Angle θ ($0^\circ \leqslant \theta \leqslant 180^\circ$) made by a line with positive x-axis	153
Slope	$m = \tan\theta$; undefined for $\theta = 90^\circ$	153

Term	Definition	Page
Slope from two points	$m = (y_2 - y_1)/(x_2 - x_1)$	153
Horizontal line	y = constant (slope 0)	160
Vertical line	x = constant (slope undefined)	160
Parallel-line test	$m_1 = m_2$	154
Perpendicular-line test	$m_1 m_2 = -1$	155
Angle formula	$\tan \theta =$	$(m_2 - m_1)/(1 + m_1 m_2)$
Point-slope form	$y - y_0 = m(x - x_0)$	161
Two-point form	$y - y_1 = ((y_2 - y_1)/(x_2 - x_1))(x - x_1)$	161
Slope-intercept form (y-int c)	$y = mx + c$	162
Slope-intercept form (x-int d)	$y = m(x - d)$	162
Intercept form	$x/a + y/b = 1$	163
General form	$Ax + By + C = 0$, $(A, B) \neq (0, 0)$	163
Slope from general form	$m = -A/B$ $(B \neq 0)$	163
x-intercept of Ax + By + C = 0	$-C/A$ $(A \neq 0)$	163
y-intercept of Ax + By + C = 0	$-C/B$ $(B \neq 0)$	163
Distance point-to-line	d =	$Ax_1 + By_1 + C$
Distance between parallel lines	d =	$C_1 - C_2$
Collinearity (slope form)	Slope(AB) = Slope(BC)	174
Distance formula	$\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$	152
Section formula (internal)	$((m x_2 + n x_1)/(m + n), (m y_2 + n y_1)/(m + n))$	152
Area of triangle	½	$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$
Concurrency of three lines	Algebraic check via determinant or substitution	174
Equation of x-axis	$y = 0$	160
Equation of y-axis	$x = 0$	160

2.3 Diagrams / processes to remember

- **Fig 9.1 (p. 151):** Plot of (6, –4) and (3, 0) — orients the recap of coordinate axes and distance.
- **Fig 9.2 (p. 153):** Inclination θ of a line with the positive x-axis; visualises $0° \leqslant \theta \leqslant 180°$ and the convention that vertical lines get $\theta = 90°$.
- **Fig 9.3(i)–(ii) (pp. 153–154):** Right-triangle construction (PMQ) used to derive $m = (y_2 - y_1)/(x_2 - x_1)$ for both acute and obtuse θ.
- **Fig 9.4 (p. 154):** Two parallel lines having equal inclinations $\alpha = \beta$, hence $m_1 = m_2$. The visual proof is one of the cleanest in coordinate geometry.
- **Fig 9.5 (p. 155):** Perpendicular lines with $\beta = \alpha + 90°$, leading to $m_1 m_2 = -1$ via $\tan(\alpha + 90°) = -\cot \alpha$.
- **Fig 9.6 (p. 156):** Angle between two intersecting lines L_1, L_2 — adjacent angles θ and ϕ with $\theta + \phi = 180°$. Exactly one of θ, ϕ is acute.
- **Fig 9.7 (p. 158):** Two possible slopes (3 and –1/3) in Example 2, illustrating why the acute-angle formula uses an absolute value — two distinct lines can both make the same angle with a given line.
- **Fig 9.8 (p. 160):** Horizontal lines $y = \pm a$ and vertical lines $x = \pm b$.
- **Fig 9.10 (p. 160):** Point-slope construction for the line through $P_0(x_0, y_0)$ with slope m.
- **Fig 9.11 (p. 161):** Two-point form via collinearity of P, P_1, P_2.
- **Fig 9.12 (p. 162):** Slope-intercept derivation through (0, c).
- **Fig 9.13 (p. 163):** Intercept form using (a, 0) and (0, b).
- **Fig 9.14 (p. 165):** Perpendicular PM from $P(x_1, y_1)$ to $Ax + By + C = 0$, with auxiliary triangle PQR used to derive $d = |Ax_1 + By_1 + C|/\sqrt{(A^2 + B^2)}$.
- **Fig 9.15 (p. 166):** Two parallel lines $y = mx + c_1$ and $y = mx + c_2$; distance taken from $A(-c_1/m, 0)$ to the second line.
- **Process — find equation of a line:** identify which data you have (point + slope ⇒ point-slope; two points ⇒ two-point; slope + intercept ⇒ slope-intercept; both intercepts ⇒ intercept). Use the matching form, then simplify to $Ax + By + C = 0$ if requested.
- **Process — distance computation:** write the line in $Ax + By + C = 0$ form, substitute the point's coordinates into the numerator, take absolute value, divide by $\sqrt{(A^2 + B^2)}$.

2.4 Common confusions / NTA trap points

- **Sign inside the modulus:** in $d = |Ax_1 + By_1 + C|/\sqrt{(A^2 + B^2)}$, forgetting the absolute value (and writing a negative distance) is the commonest trap (NCERT §9.4, p. 166).
- **Acute vs obtuse angle:** $\tan \theta = (m_2 - m_1)/(1 + m_1 m_2)$ without the modulus gives the signed tangent — only after taking |·| do you get the acute angle. NTA

distractors often drop the modulus and pick the obtuse value (NCERT §9.2.3, p. 157).

- **Vertical lines have no slope:** writing $m = \infty$ instead of "undefined" can trick statement-based questions about whether $m_1 m_2 = -1$ implies perpendicularity (this assumes both lines are non-vertical) (NCERT §9.2, p. 153; §9.2.2, p. 155).
- **Slope from general form:** for $Ax + By + C = 0$, slope is $-A/B$ (not A/B and not $-B/A$). A frequent distractor (NCERT §9.3.5 and Example 9 setup, p. 167).
- **Intercept-form sign of intercepts:** in $x/a + y/b = 1$, "a" and "b" are signed intercepts. NCERT Example 8 uses $a = -3$, $b = 2$ to give $2x - 3y + 6 = 0$ (NCERT §9.3.5, p. 163).
- **Distance between two parallel lines requires equal A, B coefficients:** if the equations are written with different leading coefficients (e.g. $3x - 4y + 7 = 0$ vs $6x - 8y + 5 = 0$), they must be rescaled to match before applying $d = |C_1 - C_2|/\sqrt{(A^2 + B^2)}$ (NCERT §9.4.1, p. 167).
- **Parallel lines have slope ratio 1, perpendicular −1.** Many students invert these. The "negative reciprocal" rule applies to perpendicular slopes, not parallel.
- **Inclination > 180°.** θ is always in $[0°, 180°]$; 200° is not a valid inclination. Reduce by ±180° if needed.
- **Mistaking the y-intercept of Ax + By + C = 0.** It is $-C/B$, not $-C$. The factor of B is easy to miss when $B \neq 1$.
- **Forgetting that horizontal lines have slope 0 (not undefined).** Undefined slope is only for vertical lines.
- **Two parallel lines coincide if intercept constants match.** The "distance 0" case occurs only when both lines are identical.
- **Wrong order of subtraction in slope formula.** Either $(y_2 - y_1)/(x_2 - x_1)$ or $(y_1 - y_2)/(x_1 - x_2)$; both give the same slope. But mixing the orders (numerator from one, denominator from the other) flips the sign.
- **Misapplying the angle formula to a horizontal-vertical pair.** When one line is horizontal ($m_1 = 0$) and the other is vertical (m_2 undefined), the formula breaks down; the angle is 90° by inspection.
- **Forgetting modulus in the perpendicular distance formula.** The distance is always $|Ax_1 + By_1 + C|/\sqrt{(A^2 + B^2)}$; the sign inside indicates which side of the line the point lies on but never reverses the magnitude.
- **Confusing "intercept" with "intercept on axis".** "x-intercept = a" means the line crosses the x-axis at (a, 0), not that the x-coordinate of any point is a.

2.5 Key formulas & theorems

Formula	Statement	NCERT page
Distance	$\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$	152

Formula	Statement	NCERT page
Midpoint	$((x_1 + x_2)/2, (y_1 + y_2)/2)$	152
Section formula (internal)	$((mx_2 + nx_1)/(m+n), (my_2 + ny_1)/(m+n))$	152
Area of triangle	½	$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$
Collinearity (area)	Area = 0	152
Slope from inclination	$m = \tan\theta$	153
Slope from two points	$(y_2 - y_1)/(x_2 - x_1)$	153
Parallelism condition	$m_1 = m_2$	154
Perpendicularity condition	$m_1 m_2 = -1$	155
Angle between lines	$\tan\theta =$	$(m_2 - m_1)/(1 + m_1m_2)$
Point-slope form	$y - y_0 = m(x - x_0)$	161
Two-point form	$y - y_1 = ((y_2 - y_1)/(x_2 - x_1))(x - x_1)$	161
Slope-intercept form	$y = mx + c$	162
Intercept form	$x/a + y/b = 1$	163
General form	$Ax + By + C = 0$	163
Slope from general form	$m = -A/B$	163
x-intercept (general)	$-C/A$	163
y-intercept (general)	$-C/B$	163
Point-to-line distance		$Ax_1 + By_1 + C$
Distance between parallel lines		$C_1 - C_2$
Slope-form parallel distance		$c_1 - c_2$
Horizontal line	$y = a$	160
Vertical line	$x = b$	160
Collinearity by slope	slope(AB) = slope(BC)	174
Sum of angles around intersection	$\theta + \phi = 180^\circ$	156
Perpendicular distance from origin		C

2.6 Solved examples (NCERT-grounded)

Example A (NCERT Example 1(a), p. 155). Slope of line through (3, −2) and (−1, 4)?

Step 1 — apply formula: $m = (4 - (-2))/(-1 - 3) = 6/(-4)$. Step 2 — simplify: $m = -3/2$. Step 3 — state: slope = **$-3/2$**.

Example B (NCERT Example 3, p. 158). The line through (–2, 6) and (4, 8) is perpendicular to the line through (8, 12) and (x, 24). Find x.

Step 1 — slope m_1: $(8 - 6)/(4 - (-2)) = 2/6 = 1/3$. Step 2 — slope m_2: $(24 - 12)/(x - 8) = 12/(x - 8)$. Step 3 — perpendicularity: $m_1 m_2 = -1 \Rightarrow (1/3) \cdot 12/(x - 8) = -1 \Rightarrow 12/(x - 8) = -3 \Rightarrow x - 8 = -4 \Rightarrow x = 4$. **Answer:** $x = 4$.

Example C (NCERT Example 5, p. 161). Equation of line through (–2, 3) with slope –4?

Step 1 — apply point-slope: $y - 3 = -4(x - (-2)) = -4(x + 2)$. Step 2 — expand: $y - 3 = -4x - 8 \Rightarrow 4x + y + 5 = 0$. Step 3 — state: **$4x + y + 5 = 0$**.

Example D (NCERT Example 9, p. 167). Distance of (3, –5) from $3x - 4y - 26 = 0$?

Step 1 — identify A, B, C: $A = 3$, $B = -4$, $C = -26$. Step 2 — plug into distance formula: numerator = $|3(3) + (-4)(-5) + (-26)| = |9 + 20 - 26| = 3$. Step 3 — denominator and divide: $\sqrt{(9 + 16)} = 5$; $d = 3/5$. **Answer:** 3/5.

Example E (NCERT Example 10, p. 167). Distance between $3x - 4y + 7 = 0$ and $3x - 4y + 5 = 0$?

Step 1 — verify parallel: coefficients of x, y match; lines are parallel. Step 2 — apply parallel-distance formula: $d = |7 - 5|/\sqrt{(9 + 16)} = 2/5$. Step 3 — state: **$d = 2/5$**.

🎯 Practice MCQs

📊 PYQ Alignment

> Straight Lines is consistently among the most frequently tested units in CUET (UG) Mathematics, typically supplying around 8–10 MCQs each year. Questions are dominated by direct application of the slope, angle and perpendicular-distance formulas, conversions between standard forms (especially extracting slope/intercepts from $Ax + By + C = 0$), perpendicularity/parallelism conditions, and distance between parallel lines — the same formula-driven items modelled in NCERT Examples 1–10.