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CUET · MATHEMATICS · CLASS XI · CODE 319

Trigonometric Functions

CUET unit: Trigonometric Functions

By **UniDrill** · NCERT-grounded study material

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📌 Snapshot

- Generalises school-level trigonometric ratios of acute angles into trigonometric functions defined for any real number using the unit circle.
- Establishes the two units of angular measurement (degree and radian), the conversion π radian = 180°, and the arc-length relation $l = r\theta$.
- Defines sin x and cos x as coordinates of a point on the unit circle, and derives the signs, domain, range and quadrant-behaviour of all six functions.
- Builds the entire system of identities — sum/difference, double-angle, triple-angle and sum-to-product/product-to-sum — from a single congruence argument on the unit circle.
- CUET tests value-computation at standard angles, identity application (especially 2A, 3A and C±D forms), sign-by-quadrant questions, and arc-length/radian conversions.
- Domain/range and period of the six functions are routine objective items every year.

📖 Detailed Notes

2.1 Core concepts

Angles and rotation. An angle is a measure of rotation of a ray about its initial point; rotation anticlockwise is positive and clockwise is negative (NCERT §3.2, p. 44). The two arms are the initial side and terminal side, meeting at the vertex.

Degree measure. A degree is 1/360 of one complete revolution; 1° = 60′ and 1′ = 60″. Angles 360°, 270°, 420°, –30°, –420° are illustrated in Fig 3.3 (NCERT §3.2.1, p. 44). Angles greater than 360° correspond to more than one complete rotation.

Radian measure. One radian is the angle subtended at the centre of a unit circle by an arc of length 1 unit; one complete revolution = 2π radian (NCERT §3.2.2, p. 45). For a circle of radius r, an arc of length l subtends an angle θ radian where $\theta = l/r$, giving **$l = r\theta$** (the most-tested formula in this chapter).

Wrapping the real line. Radian measures and real numbers correspond one-to-one via wrapping the tangent line at A around the unit circle (NCERT §3.2.3, p. 46). This is why trig functions accept any real input.

Conversion. 2π radian = 360°, hence **π radian = 180°** ; 1 radian ≈ 57°16′ and 1° ≈ 0.01746 radian (NCERT §3.2.4, p. 46). Standard table to memorise: 30° = $\pi/6$, 45° = $\pi/4$, 60° = $\pi/3$, 90° = $\pi/2$, 180° = π, 270° = $3\pi/2$, 360° = 2π.

Unit-circle definitions. For a point P(a, b) on the unit circle with ∠AOP = x radian, **cos x = a** and **sin x = b**, so $a^2 + b^2 = 1$ gives **$\cos^2x + \sin^2x = 1$** (NCERT §3.3, p. 49). The quadrantal values follow at once:

- cos 0 = 1, sin 0 = 0
- cos $\pi/2$ = 0, sin $\pi/2$ = 1
- cos π = −1, sin π = 0
- cos $3\pi/2$ = 0, sin $3\pi/2$ = −1
- cos 2π = 1, sin 2π = 0 (NCERT §3.3, p. 49–50).

Periodicity and zeros. $\sin(2n\pi + x) = \sin x$ and $\cos(2n\pi + x) = \cos x$ for any integer n. $\sin x = 0 \Leftrightarrow x = n\pi$; $\cos x = 0 \Leftrightarrow x = (2n+1)\pi/2$ (NCERT §3.3, p. 50).

Other four functions. cosec x = 1/sin x ($x \neq n\pi$); sec x = 1/cos x ($x \neq (2n+1)\pi/2$); tan x = sin x/cos x; cot x = cos x/sin x (NCERT §3.3, p. 50).

Pythagorean identities. From $\cos^2x + \sin^2x = 1$, dividing by $\cos^2x$ gives **$1 + \tan^2x = \sec^2x$**; dividing by $\sin^2x$ gives **$1 + \cot^2x = \mathrm{cosec}^2x$** (NCERT §3.3, p. 51).

Sign by quadrant (ASTC). All positive in Q-I; only sin (and cosec) positive in Q-II; only tan (and cot) positive in Q-III; only cos (and sec) positive in Q-IV (NCERT §3.3.1, p. 52). Mnemonic "All Students Take Coffee".

Range, domain, period. Since $-1 \leqslant a, b \leqslant 1$, range of sin x and cos x is [−1, 1] for all real x. Domains: sin, cos — all R; tan, sec — R minus odd multiples of $\pi/2$; cot, cosec — R minus integral multiples of π (NCERT §3.3.2, p. 52–53). Period of sin, cos, sec, cosec is 2π; period of tan, cot is π (NCERT §3.3.2, p. 54).

Sum/difference identities. From the unit-circle congruence $P_1OP_3 \cong P_2OP_4$:

- cos(x + y) = cos x cos y − sin x sin y
- cos(x − y) = cos x cos y + sin x sin y (NCERT §3.4, p. 58–59)
- sin(x ± y) = sin x cos y ± cos x sin y (NCERT §3.4, p. 59)
- tan(x ± y) = (tan x ± tan y)/(1 ∓ tan x tan y) (NCERT §3.4, p. 60)
- cot(x ± y) similar (NCERT §3.4, p. 61)

Double-angle. $\cos 2x = \cos^2x - \sin^2x = 2\cos^2x - 1 = 1 - 2\sin^2x = (1 - \tan^2x)/(1 + \tan^2x)$; $\sin 2x = 2 \sin x \cos x = 2 \tan x/(1 + \tan^2x)$; $\tan 2x = 2 \tan x/(1 - \tan^2x)$ (NCERT §3.4, p. 61–62).

Triple-angle. $\sin 3x = 3 \sin x - 4 \sin^3x$; $\cos 3x = 4 \cos^3x - 3 \cos x$; $\tan 3x = (3 \tan x - \tan^3x)/(1 - 3 \tan^2x)$ (NCERT §3.4, p. 62).

Notable historical context. Trigonometry, literally "triangle measurement", was developed by Hipparchus (~150 BCE), Indian astronomer-mathematicians Aryabhata (~500 CE) and Bhaskara II (~1150 CE), and Arab scholars al-Khwarizmi and al-Battani. The modern function-based formulation, generalising ratios in right triangles to functions on all real numbers, became standard in the 17th century with Euler's introduction of analytic trigonometry.

Standard values table. Memorise sin/cos/tan at 0, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$. Specifically: $\sin 0 = 0$, $\sin(\pi/6) = 1/2$, $\sin(\pi/4) = 1/\sqrt{2}$, $\sin(\pi/3) = \sqrt{3}/2$, $\sin(\pi/2) = 1$; cos values reversed; $\tan(\pi/4) = 1$, $\tan(\pi/3) = \sqrt{3}$, $\tan(\pi/6) = 1/\sqrt{3}$; $\tan(\pi/2)$ undefined.

Sum-to-product / product-to-sum.

- $\cos x + \cos y = 2 \cos((x+y)/2) \cos((x-y)/2)$
- $\cos x - \cos y = -2 \sin((x+y)/2) \sin((x-y)/2)$
- $\sin x + \sin y = 2 \sin((x+y)/2) \cos((x-y)/2)$
- $\sin x - \sin y = 2 \cos((x+y)/2) \sin((x-y)/2)$ (NCERT §3.4, p. 63)
- $2 \cos x \cos y = \cos(x+y) + \cos(x-y)$; $-2 \sin x \sin y = \cos(x+y) - \cos(x-y)$; $2 \sin x \cos y = \sin(x+y) + \sin(x-y)$; $2 \cos x \sin y = \sin(x+y) - \sin(x-y)$ (NCERT §3.4, p. 64).

2.2 Definitions to memorise

Term	Definition	Page
Initial side / terminal side	Ray positions before and after rotation; vertex is the pivot	44
Positive / negative angle	Anticlockwise positive; clockwise negative	44
Degree	1/360 of one complete revolution; $1^\circ = 60'$, $1' = 60''$	44
Radian	Angle subtended at centre of unit circle by arc of length 1	45
Arc-length formula	$l = r\theta$, θ in radians	45
Degree–radian conversion	π rad = 180°; 1 rad ≈ 57°16′; $1^\circ = \pi/180$ rad	46
Quadrantal angle	Integer multiple of $\pi/2$	50
sin x, cos x	y- and x-coordinate of unit-circle point at angle x	49
tan x	sin x / cos x	50
cot x	cos x / sin x	50
sec x	1/cos x	50
cosec x	1/sin x	50
Pythagorean I	$\sin^2 x + \cos^2 x = 1$	49
Pythagorean II	$1 + \tan^2 x = \sec^2 x$	51
Pythagorean III	$1 + \cot^2 x = \csc^2 x$	51
Period	Smallest $T > 0$ with $f(x + T) = f(x)$	54

Term	Definition	Page
Period of sin/cos	2π	54
Period of tan/cot	π	54
Range of sin/cos	[−1, 1]	52
Range of sec/cosec	(−∞, −1] ∪ [1, ∞)	53
Domain of tan	R −	53
Domain of cot	R −	53
Even function	cos(−x) = cos x; sec(−x) = sec x	51
Odd function	sin(−x) = −sin x; tan(−x) = −tan x	51
ASTC	Quadrant sign rule (All, Sin, Tan, Cos)	52

2.3 Diagrams / processes to remember

- **Fig 3.1, p. 44** — initial side, terminal side, vertex; sign of angle by direction.
- **Fig 3.3, p. 44** — pictorial 360°, 180°, 270°, 420°, −30°, −420°.
- **Fig 3.4(i)–(iv), p. 45** — angles of 1 rad, −1 rad, 1½ rad, −1½ rad.
- **Fig 3.5, p. 46** — wrapping of real line around unit circle.
- **Fig 3.6, p. 49** — defines P(a, b), cos x = a, sin x = b.
- **Fig 3.7, p. 51** — symmetry leading to cos(−x) = cos x, sin(−x) = −sin x.
- **Figs 3.8–3.13, p. 54–55** — graphs of sin, cos, tan, cot, sec, cosec showing periods and asymptotes.
- **Fig 3.14, p. 58** — four points P_1, P_2, P_3, P_4 used in the cos(x + y) congruence proof.

Process for sign by quadrant. Step 1 — reduce angle to its equivalent in $[0, 2\pi)$ by subtracting 2π or 360° appropriately. Step 2 — identify the quadrant. Step 3 — apply ASTC to fix sign. Step 4 — compute magnitude using the reference angle.

Process for sum-to-product simplification. Step 1 — match the expression to one of the four standard forms. Step 2 — identify x and y. Step 3 — write the product form with (x + y)/2 and (x − y)/2.

Process for general value computation. Step 1 — subtract integer multiples of period (2π for sin/cos, π for tan/cot). Step 2 — reduce to reference angle in $[0, \pi/2]$. Step 3 — apply ASTC sign. Step 4 — read off from the standard 30°/45°/60° table.

Quick C-D Identity recall. sin C + sin D = 2 sin((C+D)/2) cos((C−D)/2); sin C − sin D = 2 cos((C+D)/2) sin((C−D)/2). cos C + cos D = 2 cos((C+D)/2) cos((C−D)/2); cos C − cos D = −2 sin((C+D)/2) sin((C−D)/2). Notice only "cos − cos" carries the minus sign.

2.4 Common confusions / NTA trap points

- **Degree vs radian.** sin 30 (interpreted as 30 radians) ≠ sin 30°. Bare θ = radians (NCERT convention, p. 47).
- **$l = r\theta$ with θ in degrees.** Formula valid only with θ in radians; failing to convert produces a × $180/\pi$ factor error.
- **Sign in quadrant.** cos x is negative in Q-II, tan x negative in Q-IV — the ASTC table on p. 52 is non-negotiable.
- **Three forms of cos 2x.** $\cos^2 x - \sin^2 x$, $2\cos^2 x - 1$, $1 - 2\sin^2 x$ — NTA shuffles them as distractors.
- **Domain exclusion.** tan x and sec x undefined at odd multiples of $\pi/2$; cosec x and cot x undefined at integer multiples of π.
- **Sign in cos − cos.** cos x − cos y = −2 sin((x+y)/2) sin((x−y)/2); students drop the minus.
- **Period of sec / cosec.** Both are 2π, not π.
- **Triple angle sign.** $\sin 3x = 3\sin x - 4\sin^3 x$ but $\cos 3x = 4\cos^3 x - 3\cos x$ — the "3" and "4" swap positions.
- **Domain of $\sqrt{(\sin x)}$.** Requires $\sin x \geq 0$, i.e. $x \in [2n\pi, 2n\pi + \pi]$.
- **Sec range.** $(-\infty, -1] \cup [1, \infty)$, not $[-1, 1]$.
- **cot 0 undefined**, but tan 0 = 0 — easy to swap.
- **General solutions.** $\sin x = \sin\alpha \Rightarrow x = n\pi + (-1)^n\alpha$ is examined in Class-XII chapters, but the seed periodicity is from Class XI here.
- **Sin and cos symmetry.** sin is odd (graph symmetric about origin) and cos is even (symmetric about y-axis). Forgetting this leads to wrong signs in identity reductions.
- **Confusing degree minutes with decimal degrees.** 30°30′ is 30.5°, not 30.30°; minutes are 1/60 of a degree.
- **Treating tan as continuous at $\pi/2$.** tan is undefined at odd multiples of $\pi/2$; the graph has vertical asymptotes there.
- **Wrong choice of reference angle.** The reference angle is the acute angle between the terminal side and the x-axis (not the y-axis). Mis-identifying it flips the value.
- **Half-angle ambiguity.** $\sin(x/2) = \pm\sqrt{((1 - \cos x)/2)}$; the sign depends on the quadrant of x/2, not of x.
- **Misreading product-to-sum.** The identity 2 sin A cos B = sin(A + B) + sin(A − B) preserves sin on RHS only if the first factor is sin; with cos sin, it gives sin(A + B) − sin(A − B).
- **Forgetting that cosec, sec values lie in $(-\infty, -1] \cup [1, \infty)$.** Never expect cosec x = 0.5 — outside the range.
- **Confusing radian and degree conversion direction.** Multiply degrees by $\pi/180$ to get radians; multiply radians by $180/\pi$ to get degrees.

2.5 Key formulas & theorems

Formula / Theorem	Statement	NCERT page
Arc length	$l = r\theta$ (θ in radians)	45
Degree–radian	π rad = 180°	46
Pythagorean I	$\sin^2 x + \cos^2 x = 1$	49
Pythagorean II	$1 + \tan^2 x = \sec^2 x$	51
Pythagorean III	$1 + \cot^2 x = \operatorname{cosec}^2 x$	51
Even/odd	$\cos(-x) = \cos x$; $\sin(-x) = -\sin x$	51
$\cos(x + y)$	$\cos x \cos y - \sin x \sin y$	58
$\cos(x - y)$	$\cos x \cos y + \sin x \sin y$	59
$\sin(x + y)$	$\sin x \cos y + \cos x \sin y$	59
$\sin(x - y)$	$\sin x \cos y - \cos x \sin y$	59
$\tan(x + y)$	$(\tan x + \tan y)/(1 - \tan x \tan y)$	60
$\tan(x - y)$	$(\tan x - \tan y)/(1 + \tan x \tan y)$	60
cos 2x (form 1)	$\cos^2 x - \sin^2 x$	61
cos 2x (form 2)	$2\cos^2 x - 1$	61
cos 2x (form 3)	$1 - 2\sin^2 x$	61
cos 2x (tan form)	$(1 - \tan^2 x)/(1 + \tan^2 x)$	61
sin 2x	$2 \sin x \cos x = 2 \tan x/(1 + \tan^2 x)$	62
tan 2x	$2 \tan x/(1 - \tan^2 x)$	62
sin 3x	$3 \sin x - 4 \sin^3 x$	62
cos 3x	$4 \cos^3 x - 3 \cos x$	62
tan 3x	$(3 \tan x - \tan^3 x)/(1 - 3 \tan^2 x)$	62
C + D, C − D (sin)	sin C ± sin D formulas	63
C + D, C − D (cos)	cos C + cos D = 2 cos cos; cos C − cos D = −2 sin sin	63
Product to sum	$2 \cos x \cos y = \cos(x+y) + \cos(x-y)$	64
Half-angle	$\sin^2(x/2) = (1 - \cos x)/2$; $\cos^2(x/2) = (1 + \cos x)/2$	(Misc)

2.6 Solved examples

Example 1 — Arc length. In a circle of radius 100 cm, an arc subtends an angle 30° at the centre. Find the arc length. Step 1 — Convert: $30° = 30 \times \pi/180 = \pi/6$ rad. Step 2 — $l = r\theta = 100 \times \pi/6 = 50\pi/3$ cm. **Answer:** $50\pi/3 \approx 52.36$ cm.

Example 2 — Sign by quadrant. If sin x = 3/5 and x lies in Q-II, find tan x. Step 1 — $\cos^2 x = 1 - 9/25 = 16/25 \Rightarrow \cos x = \pm 4/5$; in Q-II cos is negative, so cos x = −4/5. Step 2 — tan x = sin x / cos x = (3/5) / (−4/5) = −3/4. **Answer:** −3/4.

Example 3 — Sum identity. Find sin 75°. Step 1 — 75° = 45° + 30°. Step 2 — sin(45° + 30°) = sin 45° cos 30° + cos 45° sin 30° = $(1/\sqrt{2})(\sqrt{3}/2) + (1/\sqrt{2})(1/2) = (\sqrt{3} + 1)/(2\sqrt{2})$. **Answer:** $(\sqrt{3} + 1)/(2\sqrt{2})$.

Example 4 — Sum-to-product. Simplify (cos 7x + cos 5x)/(sin 7x − sin 5x). Step 1 — Numerator: cos 7x + cos 5x = 2 cos 6x cos x. Step 2 — Denominator: sin 7x − sin 5x = 2 cos 6x sin x. Step 3 — Ratio = cos x / sin x = cot x. **Answer:** cot x.

Example 5 — Triple-angle. Find sin 3π/8 given cos(3π/8) value not directly known; instead evaluate cos 3π using the identity $\cos 3x = 4\cos^3 x - 3\cos x$ with x = π. Step 1 — cos π = −1. Step 2 — $\cos 3\pi = 4(-1)^3 - 3(-1) = -4 + 3 = -1$. Step 3 — Confirms cos 3π = cos(π + 2π) = cos π = −1. ✓ **Answer:** −1.

🎯 Practice MCQs

📊 PYQ Alignment

CUET (UG) Mathematics consistently draws 1–2 direct questions from this chapter each year — typically a degree/radian or arc-length conversion, one identity-based simplification (often a 2A or C±D form), and a value computation at a non-standard but reducible angle such as 31π/3 or 1710°. Solving Examples 6–22 and Exercises 3.1–3.3 of NCERT covers virtually the entire CUET question bank for this unit.

For the full PYQ archive across all Mathematics chapters, see **/pyq/mathematics**.