FREE EDITION · NOTES + 3 SAMPLE MCQS

CUET · MATHEMATICS · CLASS XII · CODE 319

Application of Derivatives

CUET unit: Application of Derivatives

By **UniDrill** · NCERT-grounded study material

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📌 Snapshot

- The derivative has four practical uses: (i) rate of change of quantities, (ii) testing whether a function is increasing/decreasing on an interval, (iii) locating local and absolute extrema via the first and second derivative tests, and (iv) reading turning points / points of inflection off f'.
- The rationalised 2026-27 edition covers only sections 6.1–6.4 (Introduction, Rate of Change, Increasing/Decreasing, Maxima and Minima with the closed-interval working rule). Tangents and normals, approximations, and the equation-of-tangent material from earlier editions are out of scope.
- CUET draws heavily here — direct rate-of-change calculations (sphere dV/dt, expanding circle dA/dt), interval-finding for increasing/decreasing functions, and optimisation word problems are repeat favourites.
- The "Working Rule" for absolute maximum/minimum on a closed interval [a, b] is the engine behind almost every closed-interval MCQ.

📖 Detailed Notes

2.1 Core concepts

- The derivative dy/dx (or f'(x)) is interpreted as the rate of change of y with respect to x, and (dy/dx) at x = x0 is the rate of change at that specific point (NCERT §6.2, p. 147).
- If x = f(t) and y = g(t) both vary with a third variable t, then by Chain Rule dy/dx = (dy/dt) / (dx/dt), provided dx/dt ≠ 0 — this is the bridge used for every "related rates" problem (NCERT §6.2, p. 147).
- dy/dx is positive if y increases as x increases and negative if y decreases as x increases (NCERT §6.2, Note p. 149).
- Marginal cost = dC/dx and marginal revenue = dR/dx are instantaneous rates of change at the current level of output, illustrated with $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$ and $R(x) = 3x^2 + 36x + 5$ (NCERT §6.2, Examples 5–6, p. 150).
- A function f is increasing on I if x1 < x2 ⇒ f(x1) < f(x2) (decreasing reverses the inequality); strict versions use strict inequalities, and "constant" means f(x) = c on I (NCERT §6.3, Definition 1, p. 152).

- First-derivative test for intervals: if f is continuous on [a, b] and differentiable on (a, b), then f is increasing on [a, b] when f'(x) > 0 on (a, b), decreasing when f'(x) < 0, and constant when f'(x) = 0 (NCERT §6.3, Theorem 1, p. 153).
- A function is increasing/decreasing at a point x0 if there exists an open interval around x0 on which it is increasing/decreasing (NCERT §6.3, Definition 2, p. 153).
- To find intervals of increase/decrease: differentiate, solve f'(x) = 0 to get critical points, partition the real line, and test the sign of f' in each subinterval — illustrated for $x^2 - 4x + 6$ (decreasing on $(-\infty, 2)$, increasing on $(2, \infty)$) and $4x^3 - 6x^2 - 72x + 30$ (NCERT §6.3, Examples 10–11, pp. 155–156).
- For maxima/minima on an interval: f has a maximum at c if $f(c) \geqslant f(x)$ for all $x \in I$, and a minimum at c if $f(c) \leqslant f(x)$ for all $x \in I$ (NCERT §6.4, Definition 3, p. 160).
- A point c is a **critical point** of f if either f'(c) = 0 or f is not differentiable at c (NCERT §6.4, Note p. 164).
- **Theorem 2:** If f has a local maximum or minimum at c, then either f'(c) = 0 or f is not differentiable at c. The converse fails — $f(x) = x^3$ has f'(0) = 0 but 0 is a point of inflection, not an extremum (NCERT §6.4, p. 164).
- **First Derivative Test (Theorem 3):** at a critical point c, if f'(x) changes from + to – as x increases through c, c is a local maximum; from – to +, local minimum; no sign change, point of inflection (NCERT §6.4, p. 164).
- **Second Derivative Test (Theorem 4):** if f'(c) = 0 and f''(c) < 0, c is a local maximum; if f'(c) = 0 and f''(c) > 0, c is a local minimum; if both are zero, the test fails and one returns to the first derivative test (NCERT §6.4, p. 166).
- Every continuous function on a closed interval [a, b] attains its absolute maximum and absolute minimum at least once on that interval (NCERT §6.4.1, Theorem 5, p. 172).
- **Working rule for absolute extrema on [a, b]:** (1) find all critical points in (a, b), (2) include the endpoints a and b, (3) evaluate f at all these points, (4) the largest value is the absolute maximum and the smallest is the absolute minimum (NCERT §6.4.1, p. 172).
- Monotonic (purely increasing or purely decreasing) functions attain their maximum and minimum at the endpoints of their domain (NCERT §6.4, p. 162).

2.2 Definitions to memorise

Term	Definition	Page
Rate of change of y w.r.t. x	dy/dx, or f'(x); at x = x0 it is f'(x0)	147
Chain rule for related rates	dy/dx = (dy/dt)/(dx/dt), if dx/dt ≠ 0	147
Marginal cost / revenue		150

Term	Definition	Page
	dC/dx and dR/dx — instantaneous rate of change of total cost/revenue with output	
Increasing on I	$x_1 < x_2$ in I ⇒ $f(x_1) < f(x_2)$ (strict inequality on the right makes it strictly increasing)	152
Decreasing on I	$x_1 < x_2$ in I ⇒ $f(x_1) > f(x_2)$ (strict version analogous)	152
Maximum value on I	$f(c) \geq f(x)$ for all $x \in I$; c is a point of maximum	160
Minimum value on I	$f(c) \leq f(x)$ for all $x \in I$; c is a point of minimum	160
Local maximum at c	$\exists\, h > 0$ such that $f(c) \geq f(x)$ for all $x \in (c - h, c + h)$, $x \neq c$	163
Local minimum at c	$\exists\, h > 0$ such that $f(c) \leq f(x)$ for all $x \in (c - h, c + h)$	163
Critical point	A point in dom(f) where $f'(c) = 0$ or f is not differentiable	164
Point of inflection	A point where $f'(c) = 0$ but f' does not change sign across c	164
Absolute (global) maximum on [a, b]	The largest value of f on the closed interval; reached by comparing f at critical points and endpoints	171
Monotonic function	A function that is either increasing on the whole interval or decreasing on the whole interval	162

2.3 Diagrams / processes to remember

- **Fig 6.1, p. 152:** parabola $y = x^2$, showing graph decreases left of origin and increases right of origin — the visual anchor for "increasing/decreasing".
- **Fig 6.2, p. 153:** the three archetypes — strictly increasing, strictly decreasing, and "neither increasing nor decreasing".
- **Fig 6.3, p. 155:** sign chart for $f(x) = x^2 - 4x + 6$ split at $x = 2$.
- **Fig 6.4, p. 155 and the table on p. 156:** sign chart for $4x^3 - 6x^2 - 72x + 30$ across $(-\infty, -2)$, $(-2, 3)$, $(3, \infty)$.
- **Fig 6.7, p. 161:** graphical illustration of maximum and minimum at a point (including a non-differentiable case).
- **Fig 6.11, p. 163:** turning points A, B, C, D on a graph — A and C are valleys (local minima), B and D are hills (local maxima).
- **Fig 6.12 and Fig 6.13, pp. 163–164:** local maximum/minimum geometry and the counter-example $y = x^3$ at $x = 0$ (point of inflection).
- **Fig 6.19, p. 172:** continuous function on a closed interval [a, d] with absolute maximum at the endpoint a and absolute minimum at the endpoint d — used to motivate the closed-interval working rule.

2.5 Key formulas & theorems

Formula	Statement	NCERT page
Rate of change	dy/dx	147
Related rates	dy/dx = (dy/dt)/(dx/dt)	147
Marginal cost	dC/dx	150
Marginal revenue	dR/dx	150
Increasing on I	f'(x) > 0 on I	153
Decreasing on I	f'(x) < 0 on I	153
Constant on I	f'(x) = 0 on I	153
Critical point	f'(c) = 0 or f' DNE at c	164
First-derivative test (max)	f' changes + to − at c	164
First-derivative test (min)	f' changes − to + at c	164
Inflection point	f'(c) = 0, no sign change	164
Second-derivative test (max)	f'(c) = 0, f''(c) < 0	166
Second-derivative test (min)	f'(c) = 0, f''(c) > 0	166
Theorem 5 (absolute extrema exist)	Continuous on [a, b]	172
Working rule (absolute)	Compare f at critical points & endpoints	172
dV/dt of sphere	$4\pi r^2 \cdot dr/dt$	148
dA/dt of circle	$2\pi r \cdot dr/dt$	149
dV/dt of cube	$3x^2 \cdot dx/dt$	148
dS/dt of cube	$12x \cdot dx/dt$	148
Local max condition	f' = 0 or DNE; sign change + to −	164
Local min condition	f' = 0 or DNE; sign change − to +	164
Monotonic function	All increasing or all decreasing	162
Marginal rate at output	Derivative at specific x	150
Slope as rate	dy/dx interpreted physically	147
Open box volume	x(L − 2x)(W − 2x)	176

2.6 Solved examples (NCERT-grounded)

Example A (NCERT Example 2, p. 148). Cube volume $V = x^3$ increases at 9 cm^3/s. Find dS/dt when x = 10.

Step 1 — relate dV/dt to dx/dt: $dV/dt = 3x^2 \cdot dx/dt \Rightarrow dx/dt = 3/x^2$. Step 2 — surface $S = 6x^2$, so $dS/dt = 12x \cdot dx/dt$: $= 12x \cdot (3/x^2) = 36/x$. Step 3 — at x = 10: dS/dt = 36/10 = **3.6 cm^2/s**.

Example B (NCERT Example 3, p. 149). Stone-and-wave: $dr/dt = 4$ cm/s; find dA/dt at $r = 10$.

Step 1 — $A = \pi r^2$: $dA/dt = 2\pi r \cdot dr/dt$. Step 2 — substitute: $2\pi(10)(4) = 80\pi$. Step 3 — answer: **80π cm²/s**.

Example C (NCERT Example 10, p. 155). Find intervals where $f(x) = x^2 - 4x + 6$ is increasing/decreasing.

Step 1 — derivative: $f'(x) = 2x - 4$. Step 2 — critical point: $f'(x) = 0 \Rightarrow x = 2$. Step 3 — sign analysis: $f' < 0$ for $x < 2$ (decreasing); $f' > 0$ for $x > 2$ (increasing). **Decreasing on $(-\infty, 2)$, increasing on $(2, \infty)$**.

Example D (NCERT Example 20, p. 167). Local extrema of $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$.

Step 1 — f': $12x(x - 1)(x + 2) = 0$ at $x = 0, 1, -2$. Step 2 — f'': $36x^2 + 24x - 24$. Evaluate: $f''(0) = -24$, $f''(1) = 36$, $f''(-2) = 72$. Step 3 — classify: $x = 0$ local max; $x = 1$ and $x = -2$ are **local minima**.

Example E (NCERT Example 27, p. 173). Absolute max/min of $f(x) = 2x^3 - 15x^2 + 36x + 1$ on $[1, 5]$.

Step 1 — $f'(x) = 6(x-2)(x-3)$: critical points $x = 2, 3$ (both in $[1, 5]$). Step 2 — evaluate f at 1, 2, 3, 5: 24, 29, 28, 56. Step 3 — pick extremes: **abs max = 56 at $x = 5$; abs min = 24 at $x = 1$**.

2.4 Common confusions / NTA trap points

- **$f'(c) = 0 \not\Rightarrow$ extremum.** Students assume every stationary point is a max or min; x^3 at $x = 0$ is the textbook counter-example. Always check the sign change of f' (or use the second derivative test).
- **"Increasing" vs "strictly increasing".** NCERT distinguishes $f(x1) \leqslant f(x2)$ (increasing) from $f(x1) < f(x2)$ (strictly increasing). NTA often slips one for the other in distractors.
- **Local vs absolute extrema.** The absolute extremum on a closed interval can occur at an endpoint, where f' need not vanish. Students who only check $f'(x) = 0$ miss it (see Theorem 6 and the working rule, p. 172).
- **Sign of dV/dt vs dV/dr.** "Volume is increasing at 9 cm³/s" gives dV/dt, not dV/dr. NTA distractors invert this regularly.
- **Second derivative test fails when $f''(c) = 0$.** Students mistakenly conclude "no extremum" — the correct response is to fall back to the first-derivative test (NCERT §6.4, Theorem 4, p. 166).

🎯 Practice MCQs

📊 PYQ Alignment

CUET (UG) 2023–25 papers have drawn 10–14 questions a year from this unit, split roughly between (i) plug-and-chug rate-of-change problems (sphere dV/dt, expanding circle dA/dt, marginal cost/revenue at a stated x), (ii) finding the interval on which a cubic or trigonometric function is increasing/decreasing, and (iii) absolute-extremum problems on closed intervals or "wire/box/tank" optimisation word problems — almost every variant is a structural copy of an example or exercise question in this chapter.