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CUET · MATHEMATICS · CLASS XII · CODE 319

Determinants

CUET unit: Determinants

By **UniDrill** · NCERT-grounded study material

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📌 Snapshot

- Defines the determinant of a square matrix of order 1, 2 and 3 and gives the expansion rule (sum of products of row/column elements with corresponding cofactors).
- Establishes the scalar property $|kA| = k^n |A|$ for an $n \times n$ matrix.
- Gives the determinant formula for the area of a triangle with given vertices and its corollary for collinear points.
- Develops minors, cofactors, adjoint and proves $A(\text{adj } A) = (\text{adj } A)A = |A| I$, leading to $A^{-1} = (1/|A|) \text{ adj } A$ whenever $|A| \neq 0$.
- Applies determinants and matrix inverse to solve a system of linear equations (matrix method $AX = B \rightarrow X = A^{-1}B$) and to decide consistency when $|A| = 0$.

📖 Detailed Notes

2.1 Core concepts

- To every square matrix $A = [a_{ij}]$ of order n, a unique number (real or complex) called its determinant $|A|$ (also det A or Δ) is associated; it can be viewed as a function $f : M \rightarrow K$ from the set of square matrices to numbers (NCERT §4.2, p. 76–77).
- Only square matrices have determinants, and $|A|$ is read as "determinant of A", not modulus of A (NCERT §4.2 Remarks, p. 77).
- For a 1×1 matrix $A = [a]$, the determinant is defined to be a itself (NCERT §4.2.1, p. 77).
- For a 2×2 matrix with entries a_{11}, a_{12}, a_{21}, a_{22}, the determinant is $a_{11} a_{22} - a_{21} a_{12}$ (NCERT §4.2.2, p. 77).
- A 3×3 determinant is evaluated by expansion along a row or column: multiply each element by $(-1)^{i+j}$ and by the 2×2 determinant obtained on deleting its row and column, then add (NCERT §4.2.3, p. 77–79).
- Expansion along any of the three rows or three columns yields the same value — six equivalent ways exist (NCERT §4.2.3, p. 80).
- For easier calculation one should expand along the row or column with the maximum number of zeros (NCERT §4.2.3 Remark (i), p. 80).

- If $A = kB$ where A and B are square matrices of the same order n, then $|A| = k^n |B|$ (illustrated for n = 2 with $|A| = 2^2 |B|$) (NCERT §4.2.3 Remark (iii), p. 80).
- Area of a triangle with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3) equals (1/2) of the determinant whose rows are $(x_i, y_i, 1)$; absolute value is taken since area is positive (NCERT §4.3, p. 82).
- Three points are collinear iff this determinant is zero (NCERT §4.3 Remark (iii), p. 82).
- The minor M_{ij} of element a_{ij} is the determinant obtained by deleting the i-th row and j-th column; for an $n \times n$ determinant ($n \geqslant 2$) the minor has order $n - 1$ (NCERT §4.4 Definition 1 and Remark, p. 84).
- The cofactor A_{ij} of a_{ij} is defined as $A_{ij} = (-1)^{(i+j)} M_{ij}$ (NCERT §4.4 Definition 2, p. 84).
- |A| equals the sum of products of elements of any row (or column) with their corresponding cofactors (NCERT §4.4, p. 85).
- If elements of a row (or column) are multiplied with cofactors of a different row (or column), the sum is zero (NCERT §4.4 Note, p. 85).
- The adjoint adj A of $A = [a_{ij}]_{n \times n}$ is the transpose of the cofactor matrix $[A_{ij}]$ (NCERT §4.5.1 Definition 3, p. 87).
- For a 2×2 matrix the adjoint can be written down directly by swapping the diagonal entries and changing the sign of the off-diagonal entries (NCERT §4.5.1 Remark, p. 88).
- Theorem 1: For any square matrix A of order n, A(adj A) = (adj A) A = |A| I, where I is the identity of order n (NCERT §4.5.1, p. 88).
- A square matrix A is singular if |A| = 0 and non-singular if $|A| \neq 0$ (NCERT §4.5.1 Definitions 4 and 5, p. 89).
- Theorem 2: If A and B are non-singular matrices of the same order, then AB and BA are also non-singular of the same order (NCERT §4.5.1, p. 89).
- Theorem 3 (Product theorem): |AB| = |A| |B| for square matrices A, B of the same order (NCERT §4.5.1, p. 89).
- For an $n \times n$ matrix, $|\text{adj } A| = |A|^{(n - 1)}$ (NCERT §4.5.1, p. 89–90).
- Theorem 4: A square matrix A is invertible iff A is non-singular; whenever $|A| \neq 0$, $A^{(-1)} = (1/|A|)$ adj A (NCERT §4.5.1, p. 90).
- A system of linear equations in three unknowns $a_i x + b_i y + c_i z = d_i$ (i = 1, 2, 3) is written in matrix form as AX = B, where A is the coefficient matrix, $X = [x\ y\ z]^T$ and $B = [d_1\ d_2\ d_3]^T$ (NCERT §4.6.1, p. 94).
- Matrix Method (Case I): if $|A| \neq 0$, $A^{(-1)}$ exists and the unique solution is $X = A^{(-1)} B$ (NCERT §4.6.1, p. 94).
- Case II (|A| = 0): compute (adj A) B. If (adj A) B ≠ O, no solution exists and the system is inconsistent; if (adj A) B = O, the system may have infinitely many solutions or none (NCERT §4.6.1, p. 94–95).

- A system is consistent if it has at least one solution and inconsistent if it has none (NCERT §4.6 definitions, p. 94).

2.2 Definitions to memorise

Term	Definition	Page
Determinant of A	A number (real/complex) associated to every square matrix A = [a_ij], denoted	A
Determinant of order 1	If A = [a], then	A
Determinant of order 2	a_11 a_22 − a_21 a_12	77
Determinant of order 3	Sum, with signs (−1)^(i+j), of each element of a chosen row/column multiplied by its 2 × 2 minor	77–79
Minor M_ij	Determinant obtained by deleting i-th row and j-th column of the determinant of order n; itself of order n − 1	84
Cofactor A_ij	A_ij = (−1)^(i+j) M_ij	84
Adjoint adj A	Transpose of the matrix of cofactors [A_ij]	87
Singular matrix	Square matrix A with	A
Non-singular matrix	Square matrix A with	A
Inverse A^(-1)	A^(-1) = (1/	A
Area of triangle	(1/2) ·	determinant of rows (x_i, y_i, 1)
Consistent system	System of equations that has at least one solution	94
Inconsistent system	System of equations whose solution does not exist	94

2.3 Diagrams / processes to remember

- The four-step expansion of a 3 × 3 determinant along R_1 — multiply a_11 by (−1)^(1+1) and its minor, a_12 by (−1)^(1+2) and its minor, a_13 by (−1)^(1+3) and its minor, then sum (NCERT §4.2.3 Steps 1–4, p. 77–78).
- The schematic showing six equivalent expansions of a 3 × 3 determinant along R_1, R_2, R_3, C_1, C_2, C_3 yielding the same value (NCERT §4.2.3, p. 78–80).
- The 2 × 2 adjoint shortcut: swap a_11 ↔ a_22 and negate a_12 and a_21 (NCERT §4.5.1 Remark, p. 88).

- The verification array A(adj A) = diag(|A|, |A|, |A|) = |A| I used to prove Theorem 1 (NCERT §4.5.1, p. 88).
- Flow for solving AX = B: compute |A|; if non-zero, find adj A → A^(-1) = (1/|A|) adj A → X = A^(-1) B (NCERT §4.6.1 Case I, p. 94).

2.5 Key formulas & theorems

Formula	Statement	NCERT page
2 × 2 determinant	$a_{11}a_{22} - a_{12}a_{21}$	77
3 × 3 expansion	Sum of element × cofactor along a row/column	78
Scalar property		kA
Area of triangle	(1/2)	det of (x_i, y_i, 1)
Collinearity	det = 0	82
Minor M_ij	Delete row i and column j	84
Cofactor A_ij	(−1)^(i+j) M_ij	84
Expansion formula		A
Cross-row/column sum	0	85
Adjoint	Transpose of cofactor matrix	87
A(adj A)		A
Singular		A
Non-singular		A
Product theorem		AB
	adj A	
Invertibility	A invertible iff non-singular	90
Inverse formula	A^{-1} = (1/	A
AX = B form	A is coefficient, X is unknowns, B is constants	94
Unique solution		A
Inconsistent		A
Indeterminate		A
det(A^{-1})	1/	A
2 × 2 adjoint shortcut	Swap diagonal, negate off-diagonal	88
Identity matrix	I_n with diagonal 1, off-diagonal 0	88

2.6 Solved examples (NCERT-grounded)

Example A (NCERT §4.2.2 Example 1, p. 77). det([[2, 4], [−1, 2]]).

Step 1 — apply formula: 2·2 – 4·(–1) = 4 + 4. Step 2 — sum: 8. Step 3 — answer: **8**.

Example B (NCERT §4.2.3, Example 4, p. 81). Compute |3A| for |A| = 4, order 3.

Step 1 — apply scalar rule: $|3A| = 3^3 |A|$. Step 2 — compute: 27 · 4. Step 3 — answer: **108**.

Example C (NCERT §4.3 Example 6, p. 83). Area of triangle with vertices (3, 8), (–4, 2), (5, 1).

Step 1 — set up determinant: (1/2)|det([[3, 8, 1], [–4, 2, 1], [5, 1, 1]])|. Step 2 — expand R_1: 3(2 – 1) – 8(–4 – 5) + 1(–4 – 10) = 3 + 72 – 14 = 61. Step 3 — divide by 2: area = **61/2** sq units.

Example D (NCERT §4.5.1 Example 12, p. 88). Find adj A for A = [[2, 3], [1, 4]].

Step 1 — swap diagonal: (4, 2). Step 2 — negate off-diagonal: –3, –1. Step 3 — assemble: adj A = **[[4, −3], [−1, 2]]**.

Example E (NCERT §4.6.1 Example 16, p. 95). Solve 2x + 5y = 1, 3x + 2y = 7 by matrix method.

Step 1 — |A| = 4 – 15 = –11: non-singular, unique solution exists. Step 2 — A^{-1} = (–1/11)[[2, –5], [–3, 2]]: compute $X = A^{-1}B$. Step 3 — multiply: X = (–1/11)[2(1) – 5(7); –3(1) + 2(7)] = (–1/11)[–33; 11] = **[3; −1]**, so x = 3, y = –1.

2.4 Common confusions / NTA trap points

- Students often read |A| as a modulus — but |A| means "determinant of A" and is signed, not absolute (NCERT §4.2 Remark (i), p. 77).
- |kA| = k|A| is a common error; the correct rule for an n × n matrix is |kA| = k^n |A| (NCERT §4.2.3 Remark (iii), p. 80).
- Cofactor and minor are confused: the cofactor carries the sign (–1)^(i+j), the minor does not (NCERT §4.4 Definitions 1–2, p. 84).
- Multiplying elements of one row with cofactors of the same row gives |A|; with cofactors of a different row, the sum is 0 — a frequent trap in MCQs (NCERT §4.4 Note, p. 85).
- When |A| = 0 students often jump to "no solution"; instead, check (adj A) B: zero → may be consistent or inconsistent, non-zero → inconsistent (NCERT §4.6.1 Case II, p. 94–95).
- Area being a positive quantity is forgotten; the absolute value of the determinant must be taken, but if area is given, both signs must be solved (NCERT §4.3 Remarks (i)–(ii), p. 82).
- Sign of the cofactor (–1)^(i+j) is checkerboard: + – +, – + –, + – +. Forgetting this sign is the single biggest error in 3×3 expansions.

🎯 Practice MCQs

📊 PYQ Alignment

Determinants appear consistently in every CUET (UG) Mathematics paper from 2023 onwards, typically supplying 4–6 direct questions and feeding several more in the matrix-application set. Common formats are: numerical evaluation of 2×2 / 3×3 determinants, $|kA| = k^n |A|$ type plug-ins, adjoint and inverse computations for 2×2 matrices, the $|\text{adj } A| = |A|^{(n-1)}$ result, area-of-triangle / collinearity determinants, and the matrix-method consistency rules for a 2- or 3-variable linear system.