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CUET · MATHEMATICS · CLASS XII · CODE 319

Integrals

CUET unit: Integrals

By **UniDrill** · NCERT-grounded study material

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📌 Snapshot

- Integration is the inverse process of differentiation; the indefinite integral $\int f(x)dx = F(x) + C$ represents a family of antiderivatives differing by a constant.
- Builds a toolkit of standard integrals (powers, exponential, logarithmic, trigonometric, inverse trigonometric) and the three big methods of integration — substitution, partial fractions, and integration by parts (ILATE rule).
- Tabulates six "Integrals of some particular functions" ($1/(x^2-a^2)$, $1/(a^2-x^2)$, $1/(x^2+a^2)$, $1/\sqrt{x^2-a^2}$, $1/\sqrt{a^2-x^2}$, $1/\sqrt{x^2+a^2}$) and three by-parts integrals ($\sqrt{x^2-a^2}$, $\sqrt{x^2+a^2}$, $\sqrt{a^2-x^2}$).
- Develops the definite integral via the area function, states both fundamental theorems of integral calculus, and shows evaluation by substitution with changed limits.
- Lists eight properties of definite integrals (P0–P7) including the king property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and the even/odd function results — directly tested in CUET.

📖 Detailed Notes

2.1 Core concepts

- An antiderivative (primitive) of f is any function F with $F'(x) = f(x)$; the family $\{F + C : C \in R\}$ forms the indefinite integral, written $\int f(x)dx = F(x) + C$, where C is the arbitrary constant of integration (NCERT §7.1–§7.2, p. 225–227).
- Two functions having the same derivative on an interval differ only by a constant — so the family $\{F + C\}$ provides every antiderivative of f (NCERT §7.2 Remark, p. 227).
- Standard integrals derived directly from known derivatives include $\int x^n\, dx = x^{n+1}/(n+1) + C$ $(n \neq -1)$, $\int \cos x\, dx = \sin x + C$, $\int \sin x\, dx = -\cos x + C$, $\int \sec^2 x\, dx = \tan x + C$, $\int e^x\, dx = e^x + C$, $\int (1/x)dx = \log|x| + C$, $\int a^x\, dx = a^x/\log a + C$ (NCERT §7.2, p. 228–229).
- Properties of indefinite integrals: (i) $d/dx[\int f(x)dx] = f(x)$ and $\int f'(x)dx = f(x) + C$; (ii) $\int [f(x)+g(x)]dx = \int f(x)dx + \int g(x)dx$; (iii) $\int k\, f(x)dx = k\int f(x)dx$, and these generalise to any finite linear combination (NCERT §7.2.1, p. 229–231).
- Integration by substitution transforms $\int f(x)dx$ via $x = g(t)$, $dx = g'(t)dt$, giving $\int f(g(t))g'(t)dt$ — chosen so that a function whose derivative also appears in the integrand becomes the new variable (NCERT §7.3.1, p. 235–236).

- Using substitution one derives $\int \tan x\, dx = \log|\sec x| + C$, $\int \cot x\, dx = \log|\sin x| + C$, $\int \sec x\, dx = \log|\sec x + \tan x| + C$, $\int \operatorname{cosec} x\, dx = \log|\operatorname{cosec} x - \cot x| + C$ (NCERT §7.3.1, p. 237–238).
- Trigonometric identities (power-reduction, product-to-sum, $\sin 3x = 3 \sin x - 4 \sin^3 x$) reduce integrals of $\cos^2 x$, $\sin^2 x \cos^3 x$, $\sin^3 x$ etc. to standard forms (NCERT §7.3.2, p. 241–242).
- Six special-type integrals (§7.4, p. 243–246): $\int dx/(x^2-a^2) = (1/2a) \log|(x-a)/(x+a)| + C$; $\int dx/(a^2-x^2) = (1/2a) \log|(a+x)/(a-x)| + C$; $\int dx/(x^2+a^2) = (1/a) \tan^{-1}(x/a) + C$; $\int dx/\sqrt{(x^2-a^2)} = \log|x + \sqrt{(x^2-a^2)}| + C$; $\int dx/\sqrt{(a^2-x^2)} = \sin^{-1}(x/a) + C$; $\int dx/\sqrt{(x^2+a^2)} = \log|x + \sqrt{(x^2+a^2)}| + C$.
- For $\int dx/(ax^2+bx+c)$ and $\int dx/\sqrt{(ax^2+bx+c)}$ complete the square $ax^2 + bx + c = a[(x + b/2a)^2 + (c/a - b^2/4a^2)]$ to reduce to standard forms; for $\int (px+q)/(ax^2+bx+c)dx$ write $px + q = A \cdot d/dx(ax^2+bx+c) + B$ and split (NCERT §7.4, p. 246–247).
- Integration by partial fractions decomposes a proper rational function $P(x)/Q(x)$ into a sum of simpler fractions; five canonical forms are tabulated in Table 7.2 for distinct linear, repeated linear, three linear, repeated-plus-linear, and linear-plus-irreducible-quadratic denominators (NCERT §7.5, Table 7.2, p. 252–253).
- Improper rational functions are first reduced by long division: $P(x)/Q(x) = T(x) + P_1(x)/Q(x)$, then $T(x)$ is integrated as a polynomial and $P_1(x)/Q(x)$ by partial fractions (NCERT §7.5, p. 252).
- Integration by parts: $\int u \cdot (dv/dx)dx = uv - \int v \cdot (du/dx)dx$, i.e., integral of (first × second) = first × ∫second − ∫(derivative of first × ∫second). Choice of "first function" follows ILATE-style guidance — when one function is a power/polynomial it is taken as first, but inverse-trig or logarithmic functions are taken as first when paired with algebraic/exponential functions (NCERT §7.6, p. 259–260).
- A useful by-parts result: $\int e^x[f(x) + f'(x)]dx = e^x f(x) + C$ (NCERT §7.6.1, p. 262–263).
- Three by-parts square-root integrals (§7.6.2, p. 264–265): $\int \sqrt{(x^2-a^2)}dx = (x/2)\sqrt{(x^2-a^2)} - (a^2/2) \log|x + \sqrt{(x^2-a^2)}| + C$; $\int \sqrt{(x^2+a^2)}dx = (x/2)\sqrt{(x^2+a^2)} + (a^2/2) \log|x + \sqrt{(x^2+a^2)}| + C$; $\int \sqrt{(a^2-x^2)}dx = (x/2)\sqrt{(a^2-x^2)} + (a^2/2) \sin^{-1}(x/a) + C$.
- The definite integral $\int_a^b f(x)dx$ has a unique numerical value — defined either as a limit of a sum or, when F is an antiderivative of continuous f on [a, b], as $F(b) - F(a)$ (NCERT §7.7, §7.8.3 Theorem 2, p. 267–268).
- First fundamental theorem of calculus: if $A(x) = \int_a^x f(x)dx$ is the area function for continuous f on [a, b], then $A'(x) = f(x)$ on [a, b] (NCERT §7.8.1–§7.8.2 Theorem 1, p. 267–268).
- Second fundamental theorem of calculus enables evaluation: $\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$ — the arbitrary constant C cancels in the subtraction (NCERT §7.8.3, p. 268).

- Evaluation by substitution for definite integrals: substitute, change the limits accordingly, then evaluate in the new variable without going back (NCERT §7.9 Note, p. 271–272).
- Properties of definite integrals (P0–P7, §7.10, p. 273–276): P0 $\int_a^b f(x)dx = \int_a^b f(t)dt$; P1 $\int_a^b f(x)dx = -\int_\beta^a f(x)dx$ and $\int_a^a = 0$; P2 $\int_a^b = \int_a^c + \int^{cb}$; P3 $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$; P4 (king property) $\int_0^a f(x)dx = \int_0^a f(a-x)dx$; P5 $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$; P6 $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$ if $f(2a-x) = f(x)$, and $= 0$ if $f(2a-x) = -f(x)$; P7 $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$ if f is even, = 0 if f is odd.

2.2 Definitions to memorise

Term	Definition	Page
Antiderivative (primitive)	A function F such that $F'(x) = f(x)$ for all x in the interval	225–226
Indefinite integral	The family $\{F(x) + C : C \in R\}$ of all antiderivatives of f, written $\int f(x)dx$	226–227
Constant of integration	The arbitrary real C in $\int f(x)dx = F(x) + C$	226
Integrand	The function f(x) appearing inside $\int f(x)dx$	227
Variable of integration	The variable x in $\int f(x)dx$	227
Integration by substitution	Replacing x by g(t) so $dx = g'(t)dt$, transforming $\int f(x)dx$ to $\int f(g(t))g'(t)dt$	235–236
Proper rational function	P(x)/Q(x) where degree(P) < degree(Q)	252
Partial fraction decomposition	Writing a proper rational function as a sum of simpler fractions per Table 7.2	253
Integration by parts	$\int u(dv/dx)dx = uv - \int v(du/dx)dx$ — integral of product as (1st × ∫2nd) − ∫(d/dx(1st) × ∫2nd)	259–260
Definite integral	$\int_a^b f(x)dx$ — unique number F(b) − F(a) when F is an antiderivative of continuous f on [a, b]	267–268
Area function A(x)	$\int_a^x f(t)dt$ — area under y = f(x) from a to x	267
First FTC	For continuous f, $A'(x) = f(x)$ where A is the area function	267–268
Second FTC	$\int_a^b f(x)dx = F(b) - F(a)$ when $F' = f$ on [a, b]	268

2.3 Diagrams / processes to remember

- **Fig 7.1 (Area function, p. 267):** Region bounded by y = f(x), the x-axis, and the ordinates x = a, x = b; the shaded portion from a to a moving point x represents $A(x) = \int_a^x f(x)dx$ — visual basis for the first FTC.

- **Table 7.1 — Symbols/Terms (p. 227):** $\int f(x)dx$, integrand, variable of integration, "integrate", "an integral of f", integration, constant of integration.
- **Table of standard formulae (p. 228–229):** 13 derivative–integral pairs covering powers, sin, cos, $\sec^2$, cosec^2, sec tan, cosec cot, the three inverse-trig forms, e^x, 1/x, a^x.
- **Table 7.2 — Partial fraction templates (p. 253):** five rational-function shapes mapped to their decomposition forms ($A/(x-a) + B/(x-b)$; $A/(x-a) + B/(x-a)^2$; three distinct linear factors; $(x-a)^2(x-b)$; linear × irreducible quadratic with Bx+C numerator).
- **Six "particular function" formulae (p. 243):** memorise the boxed list 7.4 (1)–(6) — the (1/2a) log forms, the $(1/a)\tan^{-1}$ form, the $\log|x + \sqrt{\cdots}|$ forms, and the $\sin^{-1}(x/a)$ form.
- **Three √-integrals via by-parts (p. 264–265):** $\int\sqrt{x^2-a^2}\,dx$, $\int\sqrt{x^2+a^2}\,dx$, $\int\sqrt{a^2-x^2}\,dx$ with their $(x/2)\sqrt{\cdots} + (a^2/2)(\log \text{ or } \sin^{-1})$ structure.

2.4 Common confusions / NTA trap points

- Forgetting the constant of integration C in indefinite integrals, or wrongly adding C in definite integrals where it cancels (NCERT §7.8.3 Remark, p. 268).
- Confusing $\int dx/(x^2+a^2) = (1/a)\tan^{-1}(x/a) + C$ with $\int dx/(x^2-a^2) = (1/2a)\log|(x-a)/(x+a)| + C$ — students swap signs and the 1/a vs 1/2a factor (NCERT §7.4, p. 243).
- Misreading $\int dx/\sqrt{a^2-x^2} = \sin^{-1}(x/a) + C$ as $\cos^{-1}(x/a)$; both are antiderivatives of related functions but differ in sign — only $\sin^{-1}(x/a)$ matches the standard listing (NCERT §7.4 (5), p. 244).
- Applying integration by parts with the wrong "first function" — NCERT explicitly warns $\int x\cos x\,dx$ with cos x as first gives a worse integral, so choosing the power/polynomial as first (or log/inverse-trig as first when paired with algebraic) matters (NCERT §7.6, p. 260).
- Forgetting to change limits when using substitution in a definite integral, or changing limits but then re-substituting back to x (NCERT §7.9 Note, p. 272).
- Using property P7 without first checking whether f is even or odd — e.g., $\int_{-1}^{1} \sin^5 x \cos^4 x\,dx = 0$ because the integrand is odd (NCERT Example 31, p. 277).
- Mis-applying partial fractions when the rational function is improper (degree of numerator ⩾ degree of denominator); first do polynomial long division.
- Forgetting that for $\int dx/\sqrt{x^2+a^2}$ the answer is $\log|x + \sqrt{x^2+a^2}|$, not $\sin^{-1}(x/a)$.
- Misapplying ILATE: log and inverse-trig are usually first; algebraic and exponential are usually second; trigonometric depends on context.

2.5 Key formulas & theorems

Formula	Statement	NCERT page
$\int x^n\,dx$	$x^{(n+1)}/(n+1) + C$, $n \neq -1$	228
$\int dx/x$	log	x
$\int e^x\,dx$	$e^x + C$	228
$\int a^x\,dx$	$a^x/\log a + C$	228
$\int \sin x\,dx$	$-\cos x + C$	228
$\int \cos x\,dx$	$\sin x + C$	228
$\int \sec^2 x\,dx$	$\tan x + C$	228
$\int \mathrm{cosec}^2 x\,dx$	$-\cot x + C$	228
$\int \sec x \tan x\,dx$	$\sec x + C$	228
$\int dx/(x^2 + a^2)$	$(1/a) \tan^{-1}(x/a) + C$	243
$\int dx/(x^2 - a^2)$	$(1/2a)$ log	$(x-a)/(x+a)$
$\int dx/(a^2 - x^2)$	$(1/2a)$ log	$(a+x)/(a-x)$
$\int dx/\sqrt{(a^2 - x^2)}$	$\sin^{-1}(x/a) + C$	243
$\int dx/\sqrt{(x^2 + a^2)}$	log	$x + \sqrt{(x^2 + a^2)}$
$\int dx/\sqrt{(x^2 - a^2)}$	log	$x + \sqrt{(x^2 - a^2)}$
$\int \tan x\,dx$	log	sec x
$\int \cot x\,dx$	log	sin x
$\int \sec x\,dx$	log	sec x + tan x
$\int \mathrm{cosec}\, x\,dx$	log	cosec x − cot x
Integration by parts	$\int u\,dv = uv - \int v\,du$	259
$\int e^x[f + f']\,dx$	$e^x f(x) + C$	262
$\int \sqrt{(x^2 - a^2)}\,dx$	$(x/2)\sqrt{(x^2-a^2)} - (a^2/2)$ log	$x+\sqrt{(x^2-a^2)}$
$\int \sqrt{(x^2 + a^2)}\,dx$	$(x/2)\sqrt{(x^2+a^2)} + (a^2/2)$ log	$x+\sqrt{(x^2+a^2)}$
$\int \sqrt{(a^2 - x^2)}\,dx$	$(x/2)\sqrt{(a^2-x^2)} + (a^2/2) \sin^{-1}(x/a) + C$	265
Second FTC	$\int_a^b f\,dx = F(b) - F(a)$	268
King property	$\int_0^a f\,dx = \int_0^a f(a - x)\,dx$	275
Even function	$\int_{-a}^a f = 2 \int_0^a f$	276
Odd function	$\int_{-a}^a f = 0$	276

2.6 Solved examples (NCERT-grounded)

Example A (NCERT Example 5(ii), p. 236). Compute $\int 2x \sin(x^2 + 1)\,dx$.

Step 1 — substitute $t = x^2 + 1$: $dt = 2x\,dx$. Step 2 — rewrite: $\int \sin t\,dt$. Step 3 — integrate: $-\cos t + C =$ **$-\cos(x^2 + 1) + C$**.

Example B (NCERT Example 8(i), p. 247). $\int dx/(x^2 - 16)$.

Step 1 — identify a: $a = 4$ since $a^2 = 16$. Step 2 — apply formula: $(1/(2\cdot4)) \log|(x - 4)/(x + 4)| + C$. Step 3 — simplify: **$(1/8) \log|(x - 4)/(x + 4)| + C$**.

Example C (NCERT Example 19, p. 261). $\int x e^x dx$ by parts.

Step 1 — choose $u = x$, $dv = e^x dx$: $du = dx$, $v = e^x$. Step 2 — apply by-parts: $x e^x - \int e^x dx$. Step 3 — finish: $x e^x - e^x + C =$ **$(x - 1) e^x + C$**.

Example D (NCERT Example 18, p. 261). $\int \log x\,dx$.

Step 1 — choose $u = \log x$, $dv = dx$: $du = (1/x)\,dx$, $v = x$. Step 2 — apply by-parts: $x \log x - \int (1/x)\cdot x\,dx$. Step 3 — finish: $x \log x - x + C =$ **$x(\log x - 1) + C$**.

Example E (NCERT Example 31, p. 277). Show $\int_{-1}^{1} \sin^5 x \cos^4 x\,dx = 0$.

Step 1 — check parity: $f(-x) = \sin^5(-x) \cos^4(-x) = -\sin^5 x \cdot \cos^4 x = -f(x)$. f is odd. Step 2 — apply P7(ii): odd function over symmetric interval has integral 0. Step 3 — conclude: **integral = 0**.

🎯 Practice MCQs

📊 PYQ Alignment

Integrals consistently dominates the Calculus block of CUET (UG) Mathematics, typically yielding 10–15 questions per year combining direct standard-form recall, substitution one-liners, integration by parts on log/inverse-trig pairings, partial-fraction reductions, definite-integral evaluation by FTC, and "king" property P4 / odd-function P7 shortcuts that are perfectly tailored to NTA's single-best-answer MCQ format.