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CUET · MATHEMATICS · CLASS XII · CODE 319

Inverse Trigonometric Functions

CUET unit: Inverse Trigonometric Functions

By **UniDrill** · NCERT-grounded study material

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📌 Snapshot

- Trigonometric functions are not one-one and onto on their natural domains, so to define their inverses NCERT restricts the domain so that each function becomes bijective on a chosen interval (the principal value branch).
- The six inverse trigonometric functions — $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$, $\cot^{-1}$, $\sec^{-1}$, cosec^{-1} — each have a fixed domain and principal value branch.
- Graphs of inverse trig functions are obtained by reflecting the original trig graphs in the line y = x (interchanging x and y axes).
- Key properties include $\sin(\sin^{-1}x) = x$, $\sin^{-1}(\sin x) = x$ (within principal range), and the identity that $\sin^{-1}x$ must lie in $[-\pi/2, \pi/2]$.
- CUET routinely tests principal value computation, domain/range identification, and simplification of composite expressions like $\sin^{-1}(\sin \theta)$ for θ outside the principal branch.

📖 Detailed Notes

2.1 Core concepts

- A function $f : X \to Y$ has an inverse $f^{-1} : Y \to X$ iff f is one-one and onto; trigonometric functions over their natural domains are NOT one-one (sine repeats with period 2π and oscillates between −1 and 1), so inverses are defined only after restricting domains (NCERT §2.1, p. 18). Without this restriction, "$\sin^{-1}(½)$" would be ambiguous because infinitely many angles have sine ½.
- The six basic trig functions are: $\sin : R \to [-1, 1]$, $\cos : R \to [-1, 1]$, $\tan : R - \{(2n+1)\pi/2\} \to R$, $\cot : R - \{n\pi\} \to R$, $\sec : R - \{(2n+1)\pi/2\} \to R - (-1, 1)$, $\text{cosec} : R - \{n\pi\} \to R - (-1, 1)$ (NCERT §2.2, p. 18). Each of these has a natural domain (the largest set of reals on which the function is defined) and a restricted domain on which it becomes invertible.
- Sine restricted to $[-\pi/2, \pi/2]$ is bijective onto $[-1, 1]$; its inverse $\sin^{-1} : [-1, 1] \to [-\pi/2, \pi/2]$ is called the principal value branch (NCERT §2.2, p. 19). NCERT notes that other branches like $[\pi/2, 3\pi/2]$ would also work, but $[-\pi/2, \pi/2]$ is the conventional choice because it contains 0 and is symmetric about 0.
- Cosine restricted to $[0, \pi]$ is bijective onto $[-1, 1]$; $\cos^{-1} : [-1, 1] \to [0, \pi]$ is the principal value branch of arc cosine (NCERT §2.2, p. 20–21). Note that the $\cos^{-1}$ branch is NOT symmetric about 0; it starts at 0 and runs to π.

- Cosec restricted to $[-\pi/2, \pi/2] - \{0\}$ is bijective onto $R - (-1, 1)$; $\text{cosec}^{-1} : R - (-1, 1) \to [-\pi/2, \pi/2] - \{0\}$ is the principal value branch (NCERT §2.2, p. 21–22). The point 0 is excluded because cosec is undefined at 0.
- Sec restricted to $[0, \pi] - \{\pi/2\}$ is bijective onto $R - (-1, 1)$; $\sec^{-1} : R - (-1, 1) \to [0, \pi] - \{\pi/2\}$ is the principal value branch (NCERT §2.2, p. 22–23). The point $\pi/2$ is excluded because sec is undefined there.
- Tan restricted to $(-\pi/2, \pi/2)$ is bijective onto R; $\tan^{-1} : R \to (-\pi/2, \pi/2)$ is the principal value branch (NCERT §2.2, p. 23–24). The endpoints are excluded because tan diverges to $\pm\infty$ there.
- Cot restricted to $(0, \pi)$ is bijective onto R; $\cot^{-1} : R \to (0, \pi)$ is the principal value branch (NCERT §2.2, p. 24–25). Again the endpoints are excluded because cot is undefined at 0 and π.
- The graph of any inverse trig function is obtained either by interchanging x and y axes of the original trig graph, or as the reflection of the original graph in the line $y = x$ (NCERT §2.2, Remarks (i)–(ii), p. 19–20; Figs 2.1–2.6). Reflection in $y = x$ is the standard "every inverse function" recipe and applies here too.
- A critical notational warning: $\sin^{-1} x$ is NOT $(\sin x)^{-1}$; in fact $(\sin x)^{-1} = 1/\sin x = \text{cosec } x$ (NCERT §2.2 Note (1), p. 26). The superscript -1 denotes "inverse function", not "reciprocal".
- Whenever no branch is mentioned, the principal value branch is assumed; the value of an inverse trig function that lies in this principal branch is called its principal value (NCERT §2.2 Notes (2)–(3), p. 26).
- For all suitable x: $\sin(\sin^{-1} x) = x$ for $x \in [-1, 1]$, and $\sin^{-1}(\sin x) = x$ only for $x \in [-\pi/2, \pi/2]$; similar results hold for the other inverse trig functions in their principal ranges (NCERT §2.3, p. 27).
- For x outside the principal range one must convert: e.g. $\sin^{-1}(\sin(3\pi/5)) = \sin^{-1}(\sin(\pi - 3\pi/5)) = \sin^{-1}(\sin(2\pi/5)) = 2\pi/5$, because $2\pi/5$ lies in $[-\pi/2, \pi/2]$ (NCERT Misc. Example 6, p. 30). This "reduce to principal range first" routine appears on every CUET paper.
- Standard simplifications proved in NCERT include $\sin^{-1}(2x\sqrt{1-x^2}) = 2 \sin^{-1} x$ (for $-1/\sqrt{2} \leqslant x \leqslant 1/\sqrt{2}$) and $\sin^{-1}(2x\sqrt{1-x^2}) = 2 \cos^{-1} x$ (for $1/\sqrt{2} \leqslant x \leqslant 1$) (NCERT Example 3, p. 28). The break-point $x = 1/\sqrt{2}$ (= $\sin(\pi/4)$) is critical: the double-angle identity $\sin 2\theta = 2 \sin \theta \cos \theta$ holds algebraically for all x, but the image of $2 \sin^{-1} x$ leaves the principal range of $\sin^{-1}$ at $x = 1/\sqrt{2}$.
- Other simplifications: $3 \sin^{-1} x = \sin^{-1}(3x - 4x^3)$ on $[-1/2, 1/2]$, and $3 \cos^{-1} x = \cos^{-1}(4x^3 - 3x)$ on $[1/2, 1]$ (NCERT Exercise 2.2 Q1–Q2, p. 29). These echo the triple-angle identities $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$ and $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$.
- The 2026-27 reprint drops most "Properties of Inverse Trigonometric Functions" (the long list of arc-tangent sum/difference identities); only the basic principal-branch identities and the few above remain in scope.

2.2 Definitions to memorise

Term	Definition	Page
$\sin^{-1} x$	Domain [−1, 1], principal range [−π/2, π/2]	19, 26, 32
$\cos^{-1} x$	Domain [−1, 1], principal range [0, π]	21, 26, 32
$\tan^{-1} x$	Domain R, principal range (−π/2, π/2)	24, 26, 32
$\cot^{-1} x$	Domain R, principal range (0, π)	25, 26, 32
$\sec^{-1} x$	Domain R − (−1, 1), principal range [0, π] −	23, 26, 32
$\text{cosec}^{-1} x$	Domain R − (−1, 1), principal range [−π/2, π/2] −	22, 26, 32
Principal value branch	The branch chosen so that the trig function is bijective and its inverse is single-valued	19, 26
Principal value	The value of an inverse trig function that lies in its principal value branch	26, 32
$(\sin x)^{-1}$	Equals 1/sin x = cosec x; must NOT be confused with $\sin^{-1} x$	26, 32
Reflection in y = x	Geometric operation that converts the graph of f to graph of f^{-1}	20
One-one function	$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$; necessary for invertibility	18
Onto function	Every element of codomain is image of some domain element	18
Bijective	One-one and onto; equivalent to invertible	18
Natural domain of sin	All of R	18
Restricted domain of sin	[−π/2, π/2] for invertibility	19
Natural domain of cos	All of R	18
Restricted domain of cos	[0, π] for invertibility	21
Natural domain of tan	R −	18
Restricted domain of tan	(−π/2, π/2) for invertibility	23
$\sin(\sin^{-1} x)$	Equals x for x ∈ [−1, 1]	27

Term	Definition	Page
$\sin^{-1}(\sin x)$	Equals x ONLY for $x \in [-\pi/2, \pi/2]$	27
$\cos(\cos^{-1} x)$	Equals x for $x \in [-1, 1]$	27
Asymptotes of $\tan^{-1}$	Horizontal $y = \pi/2$ and $y = -\pi/2$	24
Asymptotes of $\cot^{-1}$	Horizontal $y = 0$ and $y = \pi$	25
Two-arc-sine identity	$\sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1} x$ on $[-1/\sqrt{2}, 1/\sqrt{2}]$	28
Three-arc-sine identity	$\sin^{-1}(3x - 4x^3) = 3\sin^{-1} x$ on $[-1/2, 1/2]$	29

2.3 Diagrams / processes to remember

- **Fig 2.1 (i), (ii), (iii), p. 20** — Graphs of $y = \sin x$ and $y = \sin^{-1} x$. The dark portion of $y = \sin x$ marks the principal branch $[-\pi/2, \pi/2]$, and the $y = \sin^{-1} x$ graph is its reflection in $y = x$. Frame (iii) overlays both to show the mirror symmetry, which is the visual definition of an inverse function.
- **Fig 2.2 (i), (ii), p. 21** — Graphs of $y = \cos x$ (with $[0, \pi]$ highlighted) and $y = \cos^{-1} x$. The $\cos^{-1}$ graph is strictly decreasing, a fact many students forget.
- **Fig 2.3 (i), (ii), p. 22** — Graphs of $y = \text{cosec } x$ and $y = \text{cosec}^{-1} x$. Note the hole at 0 in both the restricted domain of cosec and the range of cosec^{-1}.
- **Fig 2.4 (i), (ii), p. 23** — Graphs of $y = \sec x$ and $y = \sec^{-1} x$. Note the hole at $\pi/2$ in both the restricted domain of sec and the range of $\sec^{-1}$.
- **Fig 2.5 (i), (ii), p. 24** — Graphs of $y = \tan x$ and $y = \tan^{-1} x$. The $\tan^{-1}$ graph is strictly increasing, with horizontal asymptotes $y = \pm\pi/2$.
- **Fig 2.6 (i), (ii), p. 25** — Graphs of $y = \cot x$ and $y = \cot^{-1} x$. The $\cot^{-1}$ graph is strictly decreasing, with horizontal asymptotes $y = 0$ and $y = \pi$.
- **Summary table on p. 26 and p. 32** — Consolidated list of all six inverse trig functions with domains and principal-value ranges; this single table is the single highest-yield piece for CUET. Reproduce it cold every morning during revision.
- **Process — principal value of $\sin^{-1}(k)$** when $|k| \leqslant 1$: (i) find $\theta \in [0, \pi/2]$ with $\sin\theta = |k|$; (ii) take principal value $= \theta$ if $k \geqslant 0$, else $-\theta$. The answer is forced into $[-\pi/2, \pi/2]$.
- **Process — principal value of $\cos^{-1}(k)$:** (i) find $\theta \in [0, \pi/2]$ with $\cos\theta = |k|$; (ii) answer $= \theta$ if $k \geqslant 0$, else $\pi - \theta$. The answer is forced into $[0, \pi]$.
- **Process — evaluation of $\sin^{-1}(\sin\alpha)$ for α outside $[-\pi/2, \pi/2]$:** reduce α modulo 2π, then if the reduced value lies in $(\pi/2, 3\pi/2)$ use $\sin\alpha = \sin(\pi - \alpha)$. Otherwise use $\sin\alpha = \sin(\alpha - 2\pi)$ to shift into $[-\pi/2, \pi/2]$.

2.4 Common confusions / NTA trap points

- Confusing $\sin^{-1} x$ with $(\sin x)^{-1} = 1/\sin x = \text{cosec } x$ (NCERT §2.2 Note 1, p. 26). NTA loves planting this as a distractor.
- For $\sin^{-1}(\sin x)$, forgetting that the answer is x ONLY if $x \in [-\pi/2, \pi/2]$; for other x you must use $\sin x = \sin(\pi - x)$ or $\sin(x - 2\pi)$ etc. to land inside the principal branch (NCERT Misc. Example 6, p. 30).
- Mis-identifying the principal range of $\sec^{-1}$ as $[-\pi/2, \pi/2] - \{0\}$ (that's cosec^{-1}) instead of $[0, \pi] - \{\pi/2\}$ (NCERT summary table, p. 26, 32).
- For $\cot^{-1}$ of a negative number, students wrongly answer a negative angle; the principal range is $(0, \pi)$, so $\cot^{-1}(-1/\sqrt{3}) = 2\pi/3$, not $-\pi/3$ (NCERT Example 2, p. 26).
- Forgetting the identity $\sin^{-1}(2x\sqrt{(1-x^2)}) = 2 \sin^{-1} x$ is only valid for $-1/\sqrt{2} \leqslant x \leqslant 1/\sqrt{2}$; outside this interval the simpler RHS is no longer in the principal range (NCERT Example 3 (i), p. 28).
- Assuming $\cos^{-1}(-x) = -\cos^{-1} x$. Actually $\cos^{-1}(-x) = \pi - \cos^{-1} x$ because $\cos^{-1}$ has range $[0, \pi]$, which forbids negative values.
- Reading "$\tan^{-1} x$" as having range $[-\pi/2, \pi/2]$; the correct range is the open interval $(-\pi/2, \pi/2)$. The endpoints are never attained.
- Writing $\sin^{-1}(2)$ as a real number; the domain of $\sin^{-1}$ is $[-1, 1]$, so $\sin^{-1}(2)$ is undefined in the real-valued setting.
- Forgetting that $\cot^{-1}$ has the same domain R as $\tan^{-1}$, not the restricted domain of cot.
- Writing the graph of $y = \sin^{-1} x$ as a reflection in the x-axis (which gives $y = -\sin x$) instead of in $y = x$.
- Misreading $\sec^{-1} x$'s range as $(0, \pi)$; the correct range excludes $\pi/2$.
- Setting $\sin^{-1}(\sin(3\pi/5)) = 3\pi/5$; the actual value is $2\pi/5$ since $3\pi/5 > \pi/2$ is outside the principal range.
- Mis-computing $\tan^{-1}(-\sqrt{3}) = -2\pi/3$ instead of $-\pi/3$; $-2\pi/3$ is outside $(-\pi/2, \pi/2)$.
- Treating the inverse trig functions as "multivalued": in the NCERT framework they are single-valued functions on the principal branch. Treating $\sin^{-1}(½)$ as both $\pi/6$ and $5\pi/6$ is a leftover bad habit from school algebra and produces wrong CUET answers.
- Confusing the convex/concave shape of inverse graphs: $y = \sin^{-1} x$ is increasing, $y = \cos^{-1} x$ is decreasing, $y = \tan^{-1} x$ is increasing, $y = \cot^{-1} x$ is decreasing. Mis-sketching produces wrong sign of derivative in Class XII calculus problems.
- Forgetting that the inverse function exists only because we restricted the domain — the inverse of the unrestricted sine function is not a function at all. CUET sometimes phrases this in conceptual MCQs.

2.5 Key formulas & theorems

Formula	Statement	NCERT page
Domain of $\sin^{-1}$	$[-1, 1]$	19
Range of $\sin^{-1}$	$[-\pi/2, \pi/2]$	19
Domain of $\cos^{-1}$	$[-1, 1]$	21
Range of $\cos^{-1}$	$[0, \pi]$	21
Domain of $\tan^{-1}$	R	24
Range of $\tan^{-1}$	$(-\pi/2, \pi/2)$	24
Domain of $\cot^{-1}$	R	25
Range of $\cot^{-1}$	$(0, \pi)$	25
Domain of $\sec^{-1}$	$R - (-1, 1)$	23
Range of $\sec^{-1}$	$[0, \pi] -$	23
Domain of cosec^{-1}	$R - (-1, 1)$	22
Range of cosec^{-1}	$[-\pi/2, \pi/2] -$	22
$\sin(\sin^{-1} x)$	x for $x \in [-1, 1]$	27
$\sin^{-1}(\sin x)$	x for $x \in [-\pi/2, \pi/2]$	27
$\cos(\cos^{-1} x)$	x for $x \in [-1, 1]$	27
$\tan(\tan^{-1} x)$	x for $x \in R$	27
$\sin^{-1}(2x\sqrt{(1-x^2)})$	$2 \sin^{-1} x$ for $x \in [-1/\sqrt{2}, 1/\sqrt{2}]$	28
$\sin^{-1}(2x\sqrt{(1-x^2)})$	$2 \cos^{-1} x$ for $x \in [1/\sqrt{2}, 1]$	28
$\sin^{-1}(3x - 4x^3)$	$3 \sin^{-1} x$ for $x \in [-1/2, 1/2]$	29
$\cos^{-1}(4x^3 - 3x)$	$3 \cos^{-1} x$ for $x \in [1/2, 1]$	29
$\cos^{-1}(-x)$	$\pi - \cos^{-1} x$	26
Notation alert	$\sin^{-1} x \neq (\sin x)^{-1}$	26
Reflection rule	Graph of f^{-1} = reflection of f in $y = x$	20
Asymptote of $\tan^{-1}$	$y = \pm\pi/2$	24
Asymptote of $\cot^{-1}$	$y = 0$ and $y = \pi$	25

2.6 Solved examples (NCERT-grounded)

Example A (NCERT Example 1, p. 26). Find principal value of $\sin^{-1}(1/2)$.

Step 1 — identify reference angle: $\sin \theta = 1/2 \Rightarrow \theta = \pi/6$ is the standard reference. Step 2 — check principal range: $\pi/6 \in [-\pi/2, \pi/2]$. ✓ Step 3 — conclude: $\sin^{-1}(1/2) = \pi/6$. **Answer:** $\pi/6$.

Example B (NCERT Example 2, p. 26). Find principal value of $\cot^{-1}(-1/\sqrt{3})$.

Step 1 — find reference angle: $\cot\theta = 1/\sqrt{3} \Rightarrow \theta = \pi/3$. Step 2 — adjust for negative input: $\cot^{-1}$ range is $(0, \pi)$; $\cot(\pi - \pi/3) = -\cot(\pi/3) = -1/\sqrt{3}$, so principal value = $\pi - \pi/3 = 2\pi/3$. Step 3 — verify range: $2\pi/3 \in (0, \pi)$. ✓ **Answer:** $2\pi/3$.

Example C (NCERT Misc. Example 6, p. 30). Evaluate $\sin^{-1}(\sin(3\pi/5))$.

Step 1 — check range: $3\pi/5 \approx 0.6\pi$ is outside $[-\pi/2, \pi/2]$. Step 2 — use $\sin\alpha = \sin(\pi - \alpha)$: $\sin(3\pi/5) = \sin(\pi - 3\pi/5) = \sin(2\pi/5)$. Step 3 — check $2\pi/5 \in [-\pi/2, \pi/2]$: Yes, $2\pi/5 = 0.4\pi < \pi/2$. So answer = $2\pi/5$. **Answer:** $2\pi/5$.

Example D (NCERT Example 3 (i), p. 28). Show $\sin^{-1}(2x\sqrt{(1 - x^2)}) = 2\sin^{-1} x$ for $x \in [-1/\sqrt{2}, 1/\sqrt{2}]$.

Step 1 — substitute $x = \sin\theta$: with $\theta \in [-\pi/4, \pi/4]$ so $2\theta \in [-\pi/2, \pi/2]$. Then $\sqrt{(1 - x^2)} = \cos\theta$. Step 2 — apply double-angle: $2x\sqrt{(1 - x^2)} = 2\sin\theta\cos\theta = \sin 2\theta$. Step 3 — take $\sin^{-1}$: $\sin^{-1}(\sin 2\theta) = 2\theta$ (since $2\theta \in [-\pi/2, \pi/2]$) $= 2\sin^{-1} x$. **QED.**

Example E (NCERT Exercise 2.1, Q14, p. 27). Find value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$.

Step 1 — evaluate $\tan^{-1}(\sqrt{3})$: $\tan(\pi/3) = \sqrt{3}$ and $\pi/3 \in (-\pi/2, \pi/2) \Rightarrow \tan^{-1}(\sqrt{3}) = \pi/3$. Step 2 — evaluate $\sec^{-1}(-2)$: $\sec\theta = -2 \Rightarrow \cos\theta = -1/2 \Rightarrow \theta = 2\pi/3 \in [0, \pi] - \{\pi/2\}$. So $\sec^{-1}(-2) = 2\pi/3$. Step 3 — subtract: $\pi/3 - 2\pi/3 = -\pi/3$. **Answer:** $-\pi/3$.

🎯 Practice MCQs

📊 PYQ Alignment

This chapter consistently contributes 6–8 MCQs per year in CUET (UG) Mathematics (subject 319). The dominant question types are direct principal-value computation (e.g. $\sin^{-1}(-1/2)$, $\cot^{-1}(-1/\sqrt{3})$), identification of principal-value branches in a match-the-following or statement-correctness format, and one-step composite evaluations such as $\cos^{-1}(\cos\theta)$ or $\sin^{-1}(\sin\theta)$ where θ lies outside the principal branch — exactly the patterns NCERT drills in Exercises 2.1, 2.2 and the Miscellaneous Exercise.