FREE EDITION · NOTES + 3 SAMPLE MCQS

CUET · MATHEMATICS · CLASS XII · CODE 319

Linear Programming

CUET unit: Linear Programming

By **UniDrill** · NCERT-grounded study material

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📌 Snapshot

- Systems of linear inequalities in two variables (studied in Class XI) solve real-life optimisation problems such as maximising profit, minimising cost or minimising use of resources.
- A Linear Programming Problem (LPP) finds the optimal value (maximum or minimum) of a linear objective function $Z = ax + by$ in decision variables x and y, subject to linear constraints and non-negativity $x \geqslant 0$, $y \geqslant 0$.
- Only the graphical method is treated, with the Corner Point Method as the central technique — supported by Theorem 1 (optimum at a vertex) and Theorem 2 (bounded region guarantees both max and min).
- Special cases studied: bounded vs unbounded feasible regions, multiple optimal solutions (every point on the joining edge is optimal), and infeasible problems (empty feasible region).
- CUET regularly tests definitions, LPP formulation from a word problem, identification of feasible region/corner points, and evaluation of Z at corners — exactly the four pillars of this chapter.

📖 Detailed Notes

2.1 Core concepts

- Problems that seek to maximise (or minimise) profit (or cost) form a general class called **optimisation problems**; a special, important sub-class is the **linear programming problem**, illustrated through the furniture-dealer example of maximising profit from tables and chairs (NCERT §12.1, p. 394).
- Only the graphical method of solving LPPs is in scope here, although other methods (e.g. simplex) exist (NCERT §12.1, p. 394).
- Formulation begins by choosing decision variables x and y (e.g. number of tables and chairs), writing the non-negative constraints $x \geqslant 0$, $y \geqslant 0$, the resource constraints (investment $2500x + 500y \leqslant 50000$ i.e. $5x + y \leqslant 100$, storage $x + y \leqslant 60$), and the objective function $Z = 250x + 75y$ (NCERT §12.2.1, pp. 395–396).
- A **Linear Programming Problem** is one concerned with finding the optimal value (maximum or minimum) of a linear function (objective function) of several variables, subject to non-negativity of the variables and a set of linear inequalities (linear

constraints); "linear" means all relations are linear, "programming" refers to the method of determining a plan of action (NCERT §12.2.1, p. 396).

- **Objective function** $Z = ax + by$ (a, b constants) must be maximised or minimised; **decision variables** are x and y; **constraints** are the linear inequalities/equations on the variables, and $x \geq 0$, $y \geq 0$ are **non-negative restrictions** (NCERT §12.2.1, pp. 396).
- The common region determined by all the constraints (including non-negativity) is the **feasible region** (solution region); regions outside it are **infeasible regions**. Every point within and on the boundary of the feasible region is a **feasible solution**, while a point outside is an **infeasible solution** (NCERT §12.2.2, pp. 397).
- An **optimal (feasible) solution** is any point in the feasible region that gives the optimal (max or min) value of the objective function (NCERT §12.2.2, p. 398).
- **Theorem 1:** if $Z = ax + by$ has an optimal value over a feasible region (convex polygon), this value must occur at a **corner point (vertex)** of the feasible region (NCERT §12.2.2, p. 398).
- **Theorem 2:** if the feasible region R is **bounded** (can be enclosed in a circle), then Z has both a maximum and a minimum on R, each occurring at a corner point (NCERT §12.2.2, p. 398). If R is unbounded, max/min may not exist, but if it exists, it must occur at a corner point.
- **Corner Point Method** (the procedure): (1) find the feasible region and its corner points by inspection or by solving the intersecting pairs of boundary equations; (2) evaluate Z at each corner — let M and m be the largest and smallest values; (3) if the region is bounded, M and m are the max and min of Z; (4) if the region is unbounded, M is the max only if the open half-plane $ax + by > M$ has no point in common with the feasible region (else no max), and similarly m is the min only if $ax + by < m$ has no point in common (else no min) (NCERT §12.2.2, pp. 399).
- **Multiple optimal solutions:** if two corner points give the same optimum (e.g. max $Z = 180$ at C(15,15) and D(0,20) for $Z = 3x + 9y$), then every point on the line segment joining them is also optimal (NCERT Example 3 + Remark, p. 401).
- **Unbounded region — no optimum example:** for $Z = -50x + 20y$ over the region defined by $2x - y \geq -5$, $3x + y \geq 3$, $2x - 3y \leq 12$, $x,y \geq 0$, the smallest corner value is –300 at (6,0), but the open half-plane $-50x + 20y < -300$ has points in common with the feasible region, so Z has no minimum (NCERT Example 4, pp. 402–403).
- **Infeasible problem:** for $Z = 3x + 2y$ with $x + y \geq 8$ and $3x + 5y \leq 15$, $x,y \geq 0$, no point satisfies all constraints simultaneously — the feasible region is empty and the LPP has no feasible solution (NCERT Example 5, p. 403).
- General features noted: (i) the feasible region of an LPP is always a **convex region**; (ii) max/min occurs at a vertex; if two vertices give the same optimum, every point on the joining segment also does (NCERT §12.2.2 Remark, p. 403).
- Historical: the first LPP was formulated in 1941 by L. Kantorovich and F. L. Hitchcock (the **transportation problem**); G. Stigler (1945) formulated the **diet problem**; G. B.

Dantzig (1947) gave the simplex method; Kantorovich and Koopmans got the 1975 Nobel Prize in economics for linear programming (NCERT Historical Note, pp. 404–405).

2.2 Definitions to memorise

Term	Definition	Page
Objective function	Linear function Z = ax + by (a, b constants) to be maximised or minimised	p. 396
Decision variables	The variables x and y in the LPP whose values are to be determined	p. 396
Constraints	Linear inequalities or equations / restrictions on the variables of an LPP	p. 396
Non-negative restrictions	The conditions $x \geqslant 0, y \geqslant 0$	p. 396
Optimisation problem	A problem that seeks to maximise or minimise a linear function subject to linear inequality constraints	p. 396
Feasible region	Common region determined by all constraints including $x, y \geqslant 0$	p. 397
Infeasible region	The region other than the feasible region	p. 397
Feasible solution	Any point within or on the boundary of the feasible region	p. 397
Infeasible solution	Any point outside the feasible region	p. 397
Optimal (feasible) solution	Any point in the feasible region giving the optimal (max/min) value of Z	p. 398
Corner point	A point in the feasible region that is the intersection of two boundary lines (a vertex)	p. 398 (footnote)
Bounded region	Feasible region that can be enclosed within a circle	p. 398 (footnote)
Unbounded region	Feasible region that extends indefinitely in some direction	p. 398 (footnote)
Linear Programming Problem (LPP)	Problem of finding the optimal value of a linear objective function subject to non-negativity and linear constraints	p. 396

2.3 Diagrams / processes to remember

- **Fig 12.1, p. 397** — feasible region OABC for the furniture dealer with corner points O(0,0), A(20,0), B(10,50), C(0,60); maximum profit Z = 6250 at B(10,50).
- **Fig 12.2, p. 400** — bounded feasible region OABC for Maximise $Z = 4x + y$ under $x + y \leqslant 50$, $3x + y \leqslant 90$; corners (0,0), (30,0), (20,30), (0,50); max Z = 120 at (30,0).

- **Fig 12.3, p. 400** — bounded triangular region ABC for Minimise $Z = 200x + 500y$ under $x + 2y \geqslant 10$, $3x + 4y \leqslant 24$; corners (0,5), (4,3), (0,6); min $Z = 2300$ at (4,3).
- **Fig 12.4, p. 401** — bounded region ABCD for Min/Max $Z = 3x + 9y$ under $x + 3y \leqslant 60$, $x + y \geqslant 10$, $x \leqslant y$; corners (0,10), (5,5), (15,15), (0,20); min 60 at (5,5); max 180 at both C(15,15) and D(0,20) — multiple optimal solutions on segment CD.
- **Fig 12.5, p. 402** — unbounded region for $Z = -50x + 20y$ under $2x - y \geqslant -5$, $3x + y \geqslant 3$, $2x - 3y \leqslant 12$; smallest corner value –300 at (6,0) is NOT the minimum because the half-plane $-50x + 20y < -300$ intersects the feasible region.
- **Fig 12.6, p. 403** — illustration of an **infeasible** problem ($Z = 3x + 2y$ under $x + y \geqslant 8$, $3x + 5y \leqslant 15$) where no feasible region exists.
- **Process — Corner Point Method (p. 399):** (1) find feasible region and corner points; (2) evaluate Z at each corner; (3) if bounded, M = max, m = min; (4) if unbounded, verify by checking if open half-plane $ax + by > M$ (or $< m$) intersects the feasible region.

2.5 Key formulas & theorems

Formula	Statement	NCERT page
Objective function	$Z = ax + by$	396
Non-negativity	$x \geqslant 0, y \geqslant 0$	396
Linear constraints	$a_1x + b_1y \lesseqgtr c_1$ etc.	396
Feasible region	Intersection of constraints	397
Convexity	Feasible region is convex	403
Theorem 1	Optimum at corner point	398
Theorem 2	Bounded ⇒ max and min both exist	398
Corner Point Method	Evaluate Z at all corners	399
Unbounded max check	$ax + by > M$ empty in feasible region	399
Unbounded min check	$ax + by < m$ empty in feasible region	399
Multiple optima	If two corners equal, all of joining segment	401
Infeasible problem	Empty feasible region	403
Bounded region	Fits inside a circle	398
Unbounded region	Extends to infinity	398
Furniture LPP	Max $Z = 250x + 75y$	396
Optimum (250, 75)	$Z = 6250$ at (10, 50)	398
Constraint $5x + y \leqslant 100$	Investment 50000	396
Storage constraint	$x + y \leqslant 60$	396
LPP definition	Find optimum of linear Z s.t. linear constraints	396

Formula	Statement	NCERT page
Convex set	Line segment between any two points lies inside	403
Vertex / corner point	Intersection of two boundary lines	398
Decision variable	x or y in the LPP	396
Diet problem	Stigler 1945	405
Simplex method	Dantzig 1947	405
Z value at origin	0	397

2.6 Solved examples (NCERT-grounded)

Example A (NCERT furniture problem, § 12.2, p. 398). Max $Z = 250x + 75y$ subject to $5x + y \leqslant 100$, $x + y \leqslant 60$, $x, y \geqslant 0$.

Step 1 — find corners: O(0,0), A(20,0), B(10,50), C(0,60). Step 2 — evaluate Z: 0, 5000, 6250, 4500. Step 3 — choose max: **Z = 6250 at B(10, 50)**.

Example B (NCERT Example 1, p. 400). Max $Z = 4x + y$ under $x + y \leqslant 50$, $3x + y \leqslant 90$, $x, y \geqslant 0$.

Step 1 — corners: (0,0), (30,0), (20,30), (0,50). Step 2 — Z values: 0, 120, 110, 50. Step 3 — max: **Z = 120 at (30, 0)**.

Example C (NCERT Example 2, p. 400). Min $Z = 200x + 500y$ under $x + 2y \geqslant 10$, $3x + 4y \leqslant 24$, $x, y \geqslant 0$.

Step 1 — corners (triangular): (0,5), (4,3), (0,6). Step 2 — Z values: 2500, 2300, 3000. Step 3 — min: **Z = 2300 at (4, 3)**.

Example D (NCERT Example 3, p. 401). Max $Z = 3x + 9y$ under $x + 3y \leqslant 60$, $x + y \geqslant 10$, $x \leqslant y$, $x, y \geqslant 0$.

Step 1 — corners: A(0,10), B(5,5), C(15,15), D(0,20). Step 2 — Z values: 90, 60, 180, 180 (multiple optima at C and D). Step 3 — conclude: **Max Z = 180** on the segment CD.

Example E (NCERT Example 5, p. 403). Min $Z = 3x + 2y$ under $x + y \geqslant 8$, $3x + 5y \leqslant 15$, $x, y \geqslant 0$.

Step 1 — sketch: $x + y \geqslant 8$ forces above the line $y = 8 - x$; $3x + 5y \leqslant 15$ forces below $3x + 5y = 15$. Step 2 — check overlap: lines intersect at $x = 25/2$, $y = -9/2$ (negative), so no feasible point with $x, y \geqslant 0$. Step 3 — conclude: **infeasible LPP — no solution exists**.

2.4 Common confusions / NTA trap points

- **"Bounded" ≠ "closed".** Boundedness means the region fits inside some circle. An unbounded region may still be closed and have corner points — but Theorem 2 does NOT guarantee a max/min there (NCERT §12.2.2, p. 398).

- **Smallest corner value ≠ minimum on unbounded regions.** NTA likes the trap of asking you to read off the smallest tabulated Z on an unbounded region without checking the half-plane condition (cf. Example 4, p. 402 where –300 is NOT the minimum).
- **Feasible region is always convex** and the optimum is at a vertex — distractors often suggest "centre" or "any interior point" can be optimal (NCERT Remark (i), p. 403).
- **Multiple optimal solutions:** if two vertices give equal optimum, the entire segment joining them is optimal — students sometimes pick "only at the midpoint" or "only at the two corners" (NCERT Remark, p. 401).
- **Infeasible vs unbounded.** An empty feasible region (Example 5) means NO feasible solution; an unbounded region MAY still have an optimum — students conflate the two (NCERT Example 5, p. 403).
- **Non-negativity $x \geqslant 0, y \geqslant 0$** is a constraint of the LPP itself; distractors sometimes label it as the objective or as a separate "definition" — it is part of the linear constraints (NCERT §12.2.1, p. 395).

🎯 Practice MCQs

📊 PYQ Alignment

> Linear Programming is a high-yield CUET (UG) Mathematics topic, typically contributing around 8 MCQs across the paper / mock cycle. NTA frequently asks (i) definitional MCQs on objective function, constraints, feasible region and corner point, (ii) Corner Point Method numerical questions where the student must identify the maximum or minimum value of Z from a small table of vertex values, (iii) recognition of multiple optimal solutions, unbounded-no-optimum and infeasible cases, and (iv) formulation-style MCQs based on short word problems modelled on the furniture-dealer setup.