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CUET · MATHEMATICS · CLASS XII · CODE 319

Matrices

CUET unit: Matrices

By **UniDrill** · NCERT-grounded study material

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📌 Snapshot

- Establishes the matrix as an ordered rectangular array of numbers or functions, fixes notation `A = [aij]m × n`, and motivates matrices as a compact tool for systems of linear equations and many applied uses (cryptography, genetics, economics, graphics) (NCERT §3.1, p. 34).
- Classifies matrices by shape and entry pattern — row, column, square, diagonal, scalar, identity, zero — and defines equality of matrices.
- Builds matrix algebra: addition, scalar multiplication, subtraction, and the conformability rule for matrix multiplication, including non-commutativity ($AB \neq BA$ in general) and the fact that $AB = O$ does not force $A = O$ or $B = O$.
- The transpose A' (also A^T) obeys four operating rules; a matrix is symmetric when $A' = A$ and skew-symmetric when $A' = -A$. Every square matrix can be written as the sum of a symmetric and a skew-symmetric part.
- Closes with invertible matrices: definition ($AB = BA = I$), uniqueness of the inverse, and the reversal law $(AB)^{-1} = B^{-1}A^{-1}$. CUET reliably tests order-finding, element computation, AB products, symmetric/skew identification, and inverse-pair recognition.

📖 Detailed Notes

2.1 Core concepts

- A matrix is an ordered rectangular array of numbers or functions; the entries are its elements, and matrices are denoted by capital letters (NCERT §3.2, p. 36).
- A matrix with m rows and n columns has order $m \times n$ and contains mn elements; the (i, j)-th element is a_{ij}, where the i-th row is $[a_{i1} \cdots a_{in}]$ and the j-th column has entries $a_{1j} \cdots a_{mj}$ (NCERT §3.2.1, p. 36).
- In this book, $A = [a_{ij}]_{m \times n}$ and only matrices with real-number or real-valued-function entries are considered (NCERT §3.2.1, Note, p. 37).
- Types of matrices: a **column matrix** has only one column (order $m \times 1$); a **row matrix** has only one row (order $1 \times n$); a **square matrix** has equal rows and columns ($m = n$); the entries $a_{11}, a_{22}, \cdots, a_{nn}$ of a square matrix form its diagonal (NCERT §3.3, pp. 39–40).
- A **diagonal matrix** is a square matrix with $b_{ij} = 0$ whenever $i \neq j$; a **scalar matrix** is a diagonal matrix whose diagonal entries are all equal ($b_{ij} = k$ for $i = j$); the **identity**

matrix In is the scalar matrix with k = 1, so aij = 1 if i = j and 0 otherwise (NCERT §3.3 (iv)–(vi), pp. 40–41).

- Every identity matrix is a scalar matrix, but a scalar matrix is an identity matrix only when k = 1 (NCERT §3.3 (vi), p. 40). A **zero (null) matrix** O has all entries 0 (NCERT §3.3 (vii), p. 41).
- Two matrices are equal iff they have the same order and aij = bij for every i, j (NCERT §3.3.1 Definition 2, p. 41).
- Matrix addition: if A and B have the same order m × n, then A + B = [aij + bij]; if orders differ, A + B is not defined (NCERT §3.4.1, p. 44).
- Scalar multiplication: kA = [k aij]; negative –A = (–1)A; difference A – B = A + (–1)B (NCERT §3.4.2, pp. 45–46).
- Properties: matrix addition is commutative (A + B = B + A), associative ((A + B) + C = A + (B + C)), has additive identity O and additive inverse –A; for scalars k, l: k(A + B) = kA + kB and (k + l)A = kA + lA (NCERT §3.4.3 & §3.4.4, pp. 46–47).
- Matrix multiplication: product AB is defined iff number of columns of A equals number of rows of B; if A is m × n and B is n × p, then C = AB is m × p with cik = Σj aij bjk (NCERT §3.4.5, p. 51).
- AB defined does not imply BA defined; both AB and BA are defined together only when A is m × n and B is n × m. Even when both exist and have the same order, generally AB ≠ BA — matrix multiplication is non-commutative (NCERT §3.4.5, pp. 52–53, Examples 13 & 14).
- If AB = O for matrices A, B, it does not follow that A = O or B = O (NCERT §3.4.5, Example 15, p. 54).
- Properties of multiplication: associative (AB)C = A(BC); distributive A(B + C) = AB + AC and (A + B)C = AC + BC; existence of multiplicative identity IA = AI = A for square A (NCERT §3.4.6, p. 54).
- Transpose: if A = [aij]m × n, then A′ (or AT) = [aji]n × m — rows and columns are interchanged (NCERT §3.5 Definition 3, p. 61).
- Properties of transpose: (A′)′ = A; (kA)′ = kA′; (A + B)′ = A′ + B′; (AB)′ = B′A′ (NCERT §3.5.1, p. 61).
- A square matrix A is **symmetric** if A′ = A (i.e., aij = aji) and **skew-symmetric** if A′ = –A (i.e., aji = –aij), which forces all diagonal entries aii = 0 (NCERT §3.6 Definitions 4 & 5, pp. 63–64).
- For any square matrix A: A + A′ is symmetric and A – A′ is skew-symmetric; consequently every square matrix can be expressed as A = ½(A + A′) + ½(A – A′), a sum of a symmetric and a skew-symmetric matrix (NCERT §3.6 Theorems 1 & 2, pp. 64–65).
- A square matrix A of order m is **invertible** if there exists a square matrix B of order m such that AB = BA = I; B is the inverse of A, denoted A^{-1} (NCERT §3.7 Definition 6, p. 68).

- Rectangular matrices have no inverse; if $B = A^{-1}$, then $A = B^{-1}$ (NCERT §3.7 Note, p. 69). The inverse, if it exists, is unique (Theorem 3, p. 69), and $(AB)^{-1} = B^{-1}A^{-1}$ for invertible A, B of the same order (Theorem 4, p. 69).

2.2 Definitions to memorise

Term	Definition	Page
Matrix	Ordered rectangular array of numbers or functions	36
Order $m \times n$	m rows and n columns (mn elements)	36
Row matrix	Has only one row, order $1 \times n$	39
Column matrix	Has only one column, order $m \times 1$	39
Square matrix	$m = n$	39
Diagonal of A	Entries $a_{11}, a_{22}, \cdots, a_{nn}$ of a square matrix	39
Diagonal matrix	Square matrix with $b_{ij} = 0$ for $i \neq j$	40
Scalar matrix	Diagonal matrix with all diagonal entries equal (k)	40
Identity matrix I_n	Scalar matrix with $k = 1$ ($a_{ii} = 1$, $a_{ij} = 0$ if $i \neq j$)	40
Zero matrix O	All entries zero	41
Equal matrices	Same order and $a_{ij} = b_{ij}$ for all i, j	41
A + B	$[a_{ij} + b_{ij}]$, requires same order	44
kA	$[k\, a_{ij}]$, k scalar	45
AB	Defined iff cols(A) = rows(B); $c_{ik} = \Sigma_j a_{ij} b_{jk}$	51
Transpose A′ (A^T)	$[a_{ji}]_{n \times m}$ — rows and columns swapped	61
Symmetric matrix	Square matrix with $A' = A$	63
Skew-symmetric matrix	Square matrix with $A' = -A$; all $a_{ii} = 0$	63–64
Invertible matrix	Square A with B such that $AB = BA = I$; $B = A^{-1}$	68

2.3 Diagrams / processes to remember

- The Radha–Fauzia–Simran notebook/pen table on p. 35 motivates the matrix as a tabular array, and the second arrangement (rows = item type) shows how the same data can be transposed.
- The quadrilateral ABCD example on p. 37 illustrates how vertices in a plane can be packed as a 2×4 column-arrangement X or a 4×2 row-arrangement Y — a concrete row–column flip you can use to remember transpose.
- The Fatima two-factory production tables on p. 43 visualise matrix addition (corresponding entries sum) and the doubled-production table on p. 44 visualises scalar multiplication.

- The Meera–Nadeem pen/storybook computation on p. 50 walks through the row-times-column rule that defines AB; the schematic (2 × 2)(2 × 1) → (2 × 1) is the canonical conformability picture.
- Example 13 (p. 53) — a 2 × 3 times 3 × 2 product giving a 2 × 2 result and the reverse giving 3 × 3 — is the standard visual for "AB and BA need not even have the same order".

2.5 Key formulas & theorems

Formula	Statement	NCERT page
Matrix order	m × n	36
Square matrix	m = n	39
Diagonal entries	$a_{11}, a_{22}, \cdots,$ a⍰⍰	39
Identity matrix I⍰	Diagonal of 1s	40
Zero matrix O	All zero entries	41
Equality	Same order, all $a_{ij} = b_{ij}$	41
Addition	$(A + B)_{ij} = a_{ij} + b_{ij}$	44
Scalar mult.	$(kA)_{ij} = k\, a_{ij}$	45
Difference	A − B = A + (−1)B	46
Multiplication conformability	cols(A) = rows(B)	51
Product entry	c_i⍰ = $\Sigma_j\ a_{ij}\ b_j$⍰	51
Non-commutativity	AB ≠ BA in general	52
Identity property	AI = IA = A	54
Distributivity	A(B + C) = AB + AC	54
AB = O not ⇒ A = O or B = O	Standard counter-example	54
Transpose	$A' = [a_{ji}]$	61
(A')'	A	61
(kA)'	k A'	61
(A + B)'	A' + B'	61
(AB)'	B' A'	61
Symmetric	A' = A	63
Skew-symmetric	$A' = -A;\ a_{ii} = 0$	63
Sum-decomposition	A = ½(A + A') + ½(A − A')	65
Invertibility	∃ B with AB = BA = I	68
Inverse uniqueness	Theorem 3	69

Formula	Statement	NCERT page
$(AB)^{-1}$	$B^{-1}A^{-1}$	69

2.6 Solved examples (NCERT-grounded)

Example A (NCERT §3.4.5 Example 12, p. 52). A = [[6, 9], [2, 3]], B = [[2, 6, 0], [7, 9, 8]]. Find AB.

Step 1 — check conformability: 2 × 2 × 2 × 3 ⇒ AB is 2 × 3. Step 2 — compute each entry: (1,1) = 6·2 + 9·7 = 75; (1,2) = 6·6 + 9·9 = 117; (1,3) = 6·0 + 9·8 = 72; (2,1) = 2·2 + 3·7 = 25; (2,2) = 2·6 + 3·9 = 39; (2,3) = 2·0 + 3·8 = 24. Step 3 — assemble: AB = **[[75, 117, 72], [25, 39, 24]]**.

Example B (NCERT §3.4.5 Example 14, p. 53). Show AB ≠ BA for A = [[1, 0], [0, −1]], B = [[0, 1], [1, 0]].

Step 1 — AB: [[1·0 + 0·1, 1·1 + 0·0], [0·0 + (−1)·1, 0·1 + (−1)·0]] = [[0, 1], [−1, 0]]. Step 2 — BA: [[0·1 + 1·0, 0·0 + 1·(−1)], [1·1 + 0·0, 1·0 + 0·(−1)]] = [[0, −1], [1, 0]]. Step 3 — compare: AB ≠ BA ⇒ **non-commutative**.

Example C (NCERT §3.4.5 Example 15, p. 54). AB = O without A = O, B = O.

Step 1 — set A = [[0, −1], [0, 2]], B = [[3, 5], [0, 0]]: both non-zero. Step 2 — compute AB: [[0·3 + (−1)·0, 0·5 + (−1)·0], [0·3 + 2·0, 0·5 + 2·0]] = [[0, 0], [0, 0]]. Step 3 — conclude: **AB = O** without either matrix being zero.

Example D (NCERT §3.6 Theorem 1, p. 64). Show A + A' is symmetric.

Step 1 — (A + A')' = A' + (A')': using (B + C)' = B' + C'. Step 2 — (A')' = A: so (A + A')' = A' + A. Step 3 — = A + A': hence **A + A' is symmetric**.

Example E (NCERT §3.7 Theorem 4, p. 69). Show $(AB)^{-1} = B^{-1}A^{-1}$ for invertible A, B.

Step 1 — verify left inverse: $(B^{-1}A^{-1})(AB) = B^{-1}(A^{-1}A)B = B^{-1}IB = I$. Step 2 — verify right inverse: $(AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1} = AIA^{-1} = I$. Step 3 — conclude: by uniqueness of the inverse, $\mathbf{(AB)^{-1} = B^{-1}A^{-1}}$.

2.4 Common confusions / NTA trap points

- "Scalar matrix vs identity matrix" — every identity is scalar, but a scalar matrix is identity only when k = 1. NTA likes to swap these in option text (NCERT §3.3 (vi), p. 40).
- "AB defined ⇒ BA defined" — false in general. Both are defined simultaneously only when A is m × n and B is n × m; in particular always defined when both are square of the same order (NCERT §3.4.5 Remark, p. 52).
- "AB ≠ BA always" — false. Diagonal matrices of the same order commute (NCERT Note, p. 53). Distractors often claim multiplication is always non-commutative.

- "AB = O ⇒ A = O or B = O" — false; the worked example with A = [[0,–1],[0,2]], B = [[3,5],[0,0]] shows AB = O with both A, B ≠ O (NCERT Example 15, p. 54).
- "Diagonal of a skew-symmetric matrix" — must be all zero. Students forget that A′ = –A forces $a_{ii} = -a_{ii} \Rightarrow a_{ii} = 0$ (NCERT §3.6 after Definition 5, p. 63).
- "(AB)′ = A′B′" — wrong; correct rule is (AB)′ = B′A′ (order reverses), and the same reversal applies to inverse: $(AB)^{-1} = B^{-1}A^{-1}$ (NCERT §3.5.1, p. 61 and Theorem 4, p. 69).

🎯 Practice MCQs

📊 PYQ Alignment

> Matrices is one of the highest-yield Algebra units on CUET (UG) Mathematics — questions appearing every year typically include order/element counting, computing AB or A^2 from given matrices, identifying symmetric/skew-symmetric matrices (and their diagonals), transpose properties such as (AB)′ = B′A′, and conceptual MCQs on conformability and AB = O. Expect roughly 10–12 MCQs per paper drawn from this chapter together with the Determinants chapter.