

CUET · MATHEMATICS · CLASS XII · CODE 319

# Probability

CUET unit: Probability

By UniDrill · NCERT-grounded study material

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## Snapshot

- Builds on the Class XI axiomatic treatment of probability (Kolmogorov) by introducing **conditional probability**  $P(E|F)$  and its consequences (NCERT §13.1, p. 406).
- Establishes the **multiplication rule**  $P(E \cap F) = P(E) \cdot P(F|E) = P(F) \cdot P(E|F)$  and extends it to three or more events (NCERT §13.3, p. 415).
- Distinguishes **independent events** (defined via probability) from **mutually exclusive events** (defined via sets), and gives the criterion  $P(E \cap F) = P(E) \cdot P(F)$  (NCERT §13.4, p. 418).
- Proves the **theorem of total probability** and **Bayes' theorem** for a partition  $\{E_1, E_2, \dots, E_n\}$ , and applies them to "reverse probability" problems — defective items, disease testing, transport mode (NCERT §13.5, pp. 423-429).
- CUET routinely tests one MCQ each on definition recall, independence check, total-probability set-ups and Bayes' inversion — making this the highest-yield single chapter in the Probability unit.

## Detailed Notes

### 2.1 Core concepts

- Probability is treated as a function on outcomes (axiomatic approach of Kolmogorov), with discrete sample spaces and the addition rule already known from Class XI (NCERT §13.1, p. 406). The non-negativity, normalisation ( $P(S) = 1$ ) and  $\sigma$ -additivity axioms are assumed throughout.
- **Conditional probability** of  $E$  given  $F$  (with  $P(F) \neq 0$ ) is defined as  $P(E|F) = P(E \cap F) / P(F)$ ; intuitively, the occurrence of  $F$  reduces the sample space from  $S$  to  $F$  and we measure  $E$  inside  $F$  (NCERT §13.2, p. 408, Definition 1). The denominator  $P(F)$  "renormalises" so that  $P(F|F) = 1$ .
- Three **properties of conditional probability** are stated and proved: (i)  $P(S|F) = P(F|F) = 1$ , (ii)  $P((A \cup B)|F) = P(A|F) + P(B|F) - P((A \cap B)|F)$  with the disjoint-case corollary  $P((A \cup B)|F) = P(A|F) + P(B|F)$ , and (iii)  $P(E'|F) = 1 - P(E|F)$  (NCERT §13.2.1, pp. 408-409). All three reduce to the usual Kolmogorov properties applied to the restricted sample space  $F$ .
- Worked tossing/dice examples (Examples 1-7) show that the conditional definition applies even when elementary outcomes are not equally likely; the tree-diagram

example with one coin then die illustrates  $P(E|F)$  for unequal-probability sample points (NCERT §13.2, pp. 409-413).

- The **multiplication theorem on probability** follows directly from the definition:  $P(E \cap F) = P(E) \cdot P(F|E) = P(F) \cdot P(E|F)$ , provided both  $P(E) \neq 0$  and  $P(F) \neq 0$  (NCERT §13.3, p. 415). It is the basic tool for "without replacement" tree problems.
- For three events the rule extends to  $P(E \cap F \cap G) = P(E) \cdot P(F|E) \cdot P(G|EF)$ , and the urn/card examples (Examples 8-9) apply it to drawing without replacement (NCERT §13.3, pp. 415-417). The general  $n$ -event chain rule has  $n$  factors.
- **Independent events** are defined by  $P(F|E) = P(F)$  and  $P(E|F) = P(E)$ ; equivalently,  $P(E \cap F) = P(E) \cdot P(F)$ . Three events  $A, B, C$  are **mutually independent** iff all pairwise products and the triple product hold (NCERT §13.4, pp. 417-418, Definitions 2-3 + Remark iv). Pairwise independence does NOT imply mutual independence.
- Independence is sharply distinguished from mutual exclusivity: "independent" is about probabilities, "mutually exclusive" is about sets; two events with nonzero probability cannot be both (NCERT §13.4, p. 418, Remark ii). This Remark is paraphrased on virtually every CUET paper.
- A **partition**  $\{E_1, E_2, \dots, E_n\}$  of  $S$  consists of pairwise disjoint, exhaustive events each of nonzero probability (NCERT §13.5.1, p. 423). The partition is the conceptual setting for both the total-probability and Bayes formulas.
- **Theorem of total probability:** for any event  $A$ ,  $P(A) = \sum_j P(E_j) \cdot P(A|E_j)$ ; proved by writing  $A = (A \cap E_1) \cup \dots \cup (A \cap E_n)$  as a disjoint union and applying the multiplication rule (NCERT §13.5.2, p. 424). The formula is the "forward direction" computation: given hypotheses, find the overall probability of  $A$ .
- **Bayes' theorem:** for a partition  $\{E_1, \dots, E_n\}$  and any event  $A$  with  $P(A) \neq 0$ ,  $P(E_i|A) = \frac{P(E_i) \cdot P(A|E_i)}{\sum_j P(E_j) \cdot P(A|E_j)}$ ; the  $E_i$  are called **hypotheses**,  $P(E_i)$  the **priori** probabilities and  $P(E_i|A)$  the **posteriori** probabilities (NCERT §13.5, pp. 425-426). The denominator is exactly the total-probability formula for  $P(A)$ ; Bayes inverts the conditioning.
- Applications worked in full: drawing a red ball from one of two bags (Ex. 16), the two-coins-in-a-box problem (Ex. 17), HIV reliability (Ex. 18), defective bolts from three machines (Ex. 19), late-doctor transport (Ex. 20), and the truth-teller die (Ex. 21) (NCERT pp. 426-430).
- A **random variable** is a real-valued function on the sample space, illustrated by counting heads in two tosses (NCERT p. 430). Expectation and variance of a discrete random variable are touched on only in passing; full development is reserved for Class XII miscellaneous exercises and beyond.
- Conditional-probability intuition: imagine the sample space "shrinking" from  $S$  to  $F$  once you learn  $F$  has occurred. The conditional probability  $P(E|F)$  measures the share of  $F$  that is also in  $E$ . When  $F = S$  (no extra information),  $P(E|S) = \frac{P(E \cap S)}{P(S)} = P(E)$ , so conditional probability generalises unconditional probability.

- A second, equivalent definition of independence: E and F are independent iff knowing F has occurred does not change the probability of E, i.e.  $P(E|F) = P(E)$ . This makes the connection between the "probability" formulation ( $P(E \cap F) = P(E) P(F)$ ) and the "information" interpretation explicit (NCERT §13.4, p. 418).
- Bayes' theorem can be re-read as a **belief update**: the prior  $P(E_i)$  is updated by multiplying by the likelihood ratio  $P(A|E_i) / P(A)$ , giving the posterior  $P(E_i|A)$ . This is the cornerstone of statistical inference.
- The "law of total probability" reading:  $P(A)$  is a weighted average of the conditional probabilities  $P(A|E_j)$ , weighted by the priors  $P(E_j)$ . When all  $P(A|E_j)$  are equal,  $P(A)$  equals that common value, regardless of the priors.
- Independence of a class of events is a **strong** condition. For n events to be mutually independent, all  $2^n - n - 1$  joint-probability relations must hold (every subset of size  $\geq 2$ ). For three events that means four conditions (three pairwise + one triple). NCERT only requires verifying these four for Class XII.
- The complement rule for independence: if E, F are independent, so are E and F', and E' and F'. This often shortens calculations: rather than computing P(at least one) directly, use  $1 - P(\text{none})$ .
- A classic Bayes' set-up not in NCERT but mirrored in Exercise 13.3: a disease has 1% prevalence; a test has 99% sensitivity and 95% specificity. Then  $P(\text{disease} | \text{positive}) \approx 0.167$ , not 0.99 — illustrating how rarity of the disease moderates the posterior, a frequent CUET-style insight.
- Random-variable terminology: a random variable X assigns a real number to each outcome of the sample space. Every later application (binomial distribution, mean and variance computations) builds on this definition.
- The two-toss example: tossing a coin twice gives  $S = \{HH, HT, TH, TT\}$ ; let X count heads. Then  $X(HH) = 2$ ,  $X(HT) = X(TH) = 1$ ,  $X(TT) = 0$ , so X takes values 0, 1, 2 with probabilities  $1/4, 1/2, 1/4$  — the simplest non-trivial random variable, used throughout statistical inference.

## 2.2 Definitions to memorise

Term	Definition	Page
Conditional probability $P(E F)$	$P(E \cap F) / P(F)$ , provided $P(F) \neq 0$	
Property 1	$P(S)$	$P(F) = P(F)$
Property 2	$P((A \cup B))$	$P(F) = P(A)$
Property 3	$P(E')$	$P(F) = 1 - P(E)$
Multiplication rule (2 events)	$P(E \cap F) = P(E) \cdot P(F)$	$P(E) = P(F) \cdot P(E)$
	$P(E \cap F \cap G) = P(E) \cdot P(F)$	$P(E) \cdot P(G)$

Term	Definition	Page
Multiplication rule (3 events)		
Independent events	$P(E \cap F) = P(E) \cdot P(F)$	418
Equivalent independence	$P(E \cap F) = P(E) \cdot P(F)$	$F) = P(E)$ and $P(F)$
Mutually independent (three events)	All three pairwise products AND triple product	418
Pairwise independent	Only pairwise products hold (weaker)	418
Mutually exclusive	$E \cap F = \emptyset$	418
Partition of S	$E_i$ pairwise disjoint, exhaustive, each $P(E_i) > 0$	423
Theorem of total probability	$P(A) = \sum_j P(E_j) \cdot P(A E_j)$	$E_j)$
Hypothesis $E_i$	One of the partition events used in Bayes	425
Prior probability	$P(E_i)$	426
Posterior probability	$P(E_i A)$	A)
Bayes' theorem	$P(E_i A) = \frac{P(E_i)P(A E_i)}{\sum_j P(E_j)P(A E_j)}$	$A) = P(E_i)P(A E_i)$
Likelihood	$P(A E_i)$ — probability of A under hypothesis $E_i$	
Random variable	Real-valued function on the sample space	430
Sample space	Set of all possible outcomes	406
Event	Subset of sample space	406
Exhaustive events	Union equals S	423
Disjoint events	Pairwise intersection empty	423
Probability axioms	$P \geq 0$ , $P(S) = 1$ , $\sigma$ -additivity	406
Conditional sample space	F replaces S when conditioning on F	408
Tree diagram	Branch-multiplication visual for sequential experiments	412

## 2.3 Diagrams / processes to remember

- **Fig 13.1 / 13.2 (p. 412):** Tree diagram for the experiment "toss a coin; if head, toss again; if tail, throw a die" — fixes the technique of multiplying along branches to get elementary-event probabilities ( $1/4$ ,  $1/4$ ,  $1/12$  each for the six die outcomes). Tree diagrams are the universal solvent for multi-stage probability problems.

- **Fig 13.3 (p. 420):** Venn diagram showing  $E = (E \cap F) \cup (E \cap F')$  used to prove that if  $E$  and  $F$  are independent then so are  $E$  and  $F'$ .
- **Fig 13.4 (p. 424):** Partition diagram of  $S$  into  $E_1, E_2, \dots, E_n$  with  $A$  cut by each  $E_i$  — the visual basis of the total-probability proof. Sketching this figure is often the fastest way to set up a Bayes problem.
- **Bolt-machine and bag-ball Bayes set-ups (pp. 426-429):** the standard table format — list  $P(E_i)$ , then  $P(A|E_i)$ , then plug into Bayes — is the template for almost every Bayes MCQ.
- **Process — conditional probability:** (i) identify the conditioning event  $F$  and check  $P(F) > 0$ , (ii) compute  $P(E \cap F)$ , (iii) divide.
- **Process — independence check:** compute  $P(E)$ ,  $P(F)$ ,  $P(E \cap F)$  separately; check whether the product equals the intersection probability. If yes, independent; if not, dependent.
- **Process — total probability:** identify a partition  $\{E_i\}$ ; tabulate  $P(E_i)$  and  $P(A|E_i)$ ; sum the products.
- **Process — Bayes:** identify the partition and the evidence  $A$ ; compute total probability denominator; divide  $P(E_i) \cdot P(A|E_i)$  by it.
- **Process — tree diagram for sequential drawing:** at each node, label branches by conditional probability; multiply along a path to get joint probability; add across paths to a leaf event.

## 2.4 Common confusions / NTA trap points

- Confusing **independent** with **mutually exclusive** — two events with nonzero probability cannot be both; NTA loves a distractor that says "if  $A$  and  $B$  are mutually exclusive they are independent" (NCERT §13.4, p. 418, Remark ii).
- Forgetting that  $P(A|B)$  requires  **$P(B) \neq 0$** ; when  $P(B) = 0$ ,  $P(A|B)$  is **not defined** (NCERT Exercise 13.1, Q16, p. 414 — option C is the correct one).
- Treating  $P(E|F)$  and  $P(F|E)$  as interchangeable — they are equal only when  $P(E) = P(F)$  (NCERT Exercise 13.1, Q17, p. 415).
- For three events, **pairwise independence does not imply mutual independence** — the triple product  $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$  must also hold (NCERT §13.4, p. 418, Remark iv).
- In Bayes' problems, mis-identifying the partition: the  $E_i$  must be **pairwise disjoint AND exhaustive AND have nonzero probability** (NCERT §13.5.1, p. 423).
- Writing the total probability formula with the wrong conditioning direction: it is  $\sum P(E_j) \cdot P(A|E_j)$ , not  $\sum P(E_j) \cdot P(E_j|A)$  (NCERT §13.5.2, p. 424).
- Forgetting that "without replacement" changes  $P(F|E)$ ; students sometimes carry over the with-replacement probability and over-count favourable outcomes.

- Mis-stating Property 2 by dropping the  $-P((A \cap B)|F)$  term; valid only when A, B are disjoint within F.
- Confusing prior and posterior:  $P(E_i)$  is prior (before seeing A),  $P(E_i|A)$  is posterior (after seeing A). They are equal only when A is independent of  $E_i$ .
- Mis-applying Bayes to non-partitions; if the  $E_i$  overlap or are not exhaustive, the denominator does not equal  $P(A)$ .
- Using addition rule  $P(A \cup B) = P(A) + P(B)$  without subtracting  $P(A \cap B)$ , forgetting the inclusion-exclusion correction (a Class XI carryover error).
- Forgetting that mutually exclusive events with  $P(A), P(B) > 0$  satisfy  $P(A \cap B) = 0 \neq P(A) \cdot P(B)$ , hence cannot be independent.
- Forgetting that conditional probability is a probability (lies in  $[0, 1]$ ); negative or  $> 1$  answers signal arithmetic error.

## 2.5 Key formulas & theorems

Formula	Statement	NCERT page
Conditional probability	$P(E F)$	$F) = P(E \cap F)/P(F)$
Symmetric form	$P(E F)$	$F) \cdot P(F) = P(E \cap F) = P(F E) \cdot P(E)$
Multiplication rule	$P(E \cap F) = P(E) P(F E)$	$F)$
Three-event chain	$P(E \cap F \cap G) = P(E) P(F E) P(G E \cap F)$	$E) P(G E \cap F)$
Independence	$P(E \cap F) = P(E) P(F)$	418
Property 1	$P(F F)$	$F) = 1$
Property 2	$P((A \cup B) F)$	$F) = P(A F) + P(B F) - P((A \cap B) F)$
Property 3	$P(E' F)$	$F) = 1 - P(E F)$
Disjoint corollary	$A \cap B = \emptyset \Rightarrow P((A \cup B) F) = P(A F) + P(B F)$	$F) = P(A F) + P(B F)$
Total probability	$P(A) = \sum P(E_j) P(A E_j)$	$E_j)$
Bayes' theorem	$P(E_i A)$	$A) = P(E_i)P(A E_i) / \sum P(E_j)P(A E_j)$
Partition property	$\cup E_i = S; E_i \cap E_j = \emptyset (i \neq j)$	423
Independence of complements	$E, F \text{ indep} \Rightarrow E, F' \text{ indep}$	420
Triple independence	$P(A \cap B \cap C) = P(A)P(B)P(C)$ and all pairs	418
Mutually exclusive	$E \cap F = \emptyset$	418
Empty intersection probability	$P(E \cap F) = 0$ if disjoint	418
Addition rule	$P(E \cup F) = P(E) + P(F) - P(E \cap F)$	406
Disjoint addition	$E \cap F = \emptyset \Rightarrow P(E \cup F) = P(E) + P(F)$	406

Formula	Statement	NCERT page
Complement rule	$P(E') = 1 - P(E)$	406
Random variable definition	$X : S \rightarrow R$	430
Two-coin head count	$X \in$	430
Bayes likelihood ratio	$P(E_i$	$A) \propto P(E_i) P(A$
Prior probability	$P(E_i)$	426
Posterior probability	$P(E_i$	$A)$
Conditioning monotonicity	$E \subset F \Rightarrow P(E$	$F) = P(E)/P(F)$

## 2.6 Solved examples (NCERT-grounded)

**Example A (NCERT Exercise 13.1 Q1, p. 413).**  $P(E) = 0.6$ ,  $P(F) = 0.3$ ,  $P(E \cap F) = 0.2$ . Find  $P(E|F)$  and  $P(F|E)$ .

**Step 1 — apply definition:**  $P(E|F) = P(E \cap F)/P(F) = 0.2/0.3 = 2/3$ . **Step 2 — apply other direction:**  $P(F|E) = 0.2/0.6 = 1/3$ . **Step 3 — observe asymmetry:**  $P(E|F) \neq P(F|E)$ .

**Answer:**  $2/3$  and  $1/3$ .

**Example B (NCERT Example 9, p. 416).** Two cards drawn successively without replacement from a 52-card pack. Probability both are kings?

**Step 1 — P(first king):**  $4/52$ . **Step 2 — P(second king | first king):**  $3/51$ . **Step 3 — multiply:**  $(4/52)(3/51) = 12/2652 = 1/221$ . **Answer:**  $1/221$ .

**Example C (NCERT Example 10, p. 419).** Die thrown;  $E =$  "multiple of 3",  $F =$  "even". Are  $E$ ,  $F$  independent?

**Step 1 — list sets:**  $E = \{3, 6\}$ ,  $F = \{2, 4, 6\}$ ,  $E \cap F = \{6\}$ . **Step 2 — compute probabilities:**  $P(E) = 2/6 = 1/3$ ;  $P(F) = 3/6 = 1/2$ ;  $P(E \cap F) = 1/6$ . **Step 3 — check product:**  $P(E) \cdot P(F) = (1/3)(1/2) = 1/6 = P(E \cap F)$ . **Answer:** Independent.

**Example D (NCERT Example 15, p. 424–425).**  $P(\text{strike}) = 0.65$ ;  $P(\text{complete} | \text{strike}) = 0.32$ ;  $P(\text{complete} | \text{no strike}) = 0.80$ .  $P(\text{complete})$ ?

**Step 1 — partition:**  $E_1 = \text{strike}$ ,  $E_2 = \text{no strike}$ ;  $P(E_1) = 0.65$ ,  $P(E_2) = 0.35$ . **Step 2 — apply total probability:**  $P(A) = 0.65(0.32) + 0.35(0.80) = 0.208 + 0.280 = 0.488$ . **Step 3 — state:**  $P(\text{complete on time}) = 0.488$ . **Answer:**  $0.488$ .

**Example E (NCERT Example 19, pp. 428–429).** Machines A, B, C produce 25%, 35%, 40% of bolts; defective rates 5%, 4%, 2%. Given a defective,  $P(\text{from B})$ ?

**Step 1 — priors and likelihoods:**  $P(A) = 0.25$ ,  $P(B) = 0.35$ ,  $P(C) = 0.40$ ;  $P(D|A) = 0.05$ ,  $P(D|B) = 0.04$ ,  $P(D|C) = 0.02$ . **Step 2 — total probability of defective:**  $P(D) = 0.0125 + 0.0140 + 0.0080 = 0.0345$ . **Step 3 — apply Bayes:**  $P(B|D) = (0.35 \cdot 0.04)/0.0345 = 0.0140/0.0345 = 28/69$ . **Answer:**  $28/69$ .



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**Example F (NCERT Example 17, p. 427).** A box has 3 coins: one two-headed, one biased ( $P(H) = 0.75$ ), one fair. A coin is picked at random and tossed; it shows head. What is the probability it is the two-headed coin?

**Step 1 — partition:**  $E_1$  = two-headed,  $E_2$  = biased,  $E_3$  = fair; each prior =  $1/3$ . Likelihoods  $P(H|E_1) = 1$ ,  $P(H|E_2) = 3/4$ ,  $P(H|E_3) = 1/2$ . **Step 2 — total probability of head:**  $P(H) = (1/3)(1 + 3/4 + 1/2) = (1/3)(9/4) = 3/4$ . **Step 3 — Bayes:**  $P(E_1|H) = (1/3 \cdot 1)/(3/4) = 4/9$ . **Answer:**  $4/9$ .

## Practice MCQs

## PYQ Alignment

In CUET (UG) Mathematics 2023-2025, Probability has consistently contributed 4-6 MCQs per paper, with at least one direct Bayes' theorem application (bag-and-ball or machine-defective set-up), one independence-check or definition recall, and one conditional probability computation. Statement-based and assertion-reason items on the difference between independent and mutually exclusive events recur almost every year, making the precise wording of NCERT §13.4 Remarks (ii) and (iv) essential to memorise.