FREE EDITION · NOTES + 3 SAMPLE MCQS

CUET · MATHEMATICS · CLASS XII · CODE 319

Relations and Functions

CUET unit: Relations and Functions

By **UniDrill** · NCERT-grounded study material

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📌 Snapshot

- Extends the Class XI idea of "relation" (subset of A x A) and "function" by classifying relations as empty, universal, reflexive, symmetric, transitive and equivalence — and by classifying functions as injective (one-one), surjective (onto) and bijective.
- Any equivalence relation partitions a set into mutually disjoint equivalence classes.
- Develops composition of functions (gof) and uses it to define an invertible function via the conditions gof = I_X and fog = I_Y.
- Establishes the key CUET-friendly result: a function is invertible iff it is a bijection — and the characteristic property that for a finite set X, a self-map f : X to X is one-one iff it is onto.
- Highly examinable in CUET because every concept here generates short, definition-based MCQs and concrete f(x) examples.

📖 Detailed Notes

2.1 Core concepts

- A relation R in a set A is any subset of A x A; the two extreme relations are the empty relation R = phi and the universal relation R = A x A — both called trivial relations (NCERT §1.2, p. 2). Relations encode any binary "is related to" structure on a set.
- Empty relation example: in A = {1,2,3,4}, R = {(a,b) : a - b = 10} contains no pair, so R = phi (NCERT §1.2, p. 2).
- Universal relation example: in the same A, R' = {(a,b) : |a - b| >= 0} equals A x A (NCERT §1.2, p. 2). Both extremes are technically equivalence relations.
- A relation R in A is reflexive if (a,a) belongs to R for every a in A; symmetric if (a1,a2) in R implies (a2,a1) in R; transitive if (a1,a2) in R and (a2,a3) in R imply (a1,a3) in R (NCERT §1.2, p. 2).
- A relation that is simultaneously reflexive, symmetric and transitive is called an equivalence relation (NCERT §1.2, p. 3). Equivalence relations capture the idea of "indistinguishable up to some criterion".

- "Congruence of triangles" on the set T of all triangles in a plane is the standard equivalence relation example, since every triangle is congruent to itself and congruence is symmetric and transitive (NCERT §1.2, Example 2, p. 3).
- "L1 is perpendicular to L2" on the set L of lines is symmetric but neither reflexive nor transitive — a line cannot be perpendicular to itself, and two perpendiculars to a common line are parallel, not perpendicular (NCERT §1.2, Example 3, p. 3). This is the standard "fails some properties" example.
- Every equivalence relation R on a set X partitions X into mutually disjoint equivalence classes A_i such that elements of A_i are related to each other, no element of A_i is related to an element of A_j (i != j), and union of A_i = X (NCERT §1.2, p. 4). The reverse is also true: every partition defines an equivalence relation.
- For R = {(a,b) : 2 divides a - b} on Z, equivalence classes are [0] = set of even integers and [1] = set of odd integers, and [0] = [2r], [1] = [2r+1] for r in Z (NCERT §1.2, p. 4).
- For R = {(a,b) : 3 divides a - b} on Z, the three equivalence classes are [0] = multiples of 3, [1] = numbers of form 3r + 1 and [2] = numbers of form 3r + 2 (NCERT §1.2, p. 4). In general, "n divides a – b" gives n equivalence classes — the residue classes mod n.
- A function f : X to Y is one-one (injective) if distinct elements of X have distinct images, i.e., f(x1) = f(x2) implies x1 = x2; otherwise it is many-one (NCERT §1.3, Definition 5, p. 7).
- A function f : X to Y is onto (surjective) if every y in Y is the image of some x in X — equivalently, Range of f = Y (NCERT §1.3, Definition 6 & Remark, pp. 7–8).
- f is bijective if it is both one-one and onto (NCERT §1.3, Definition 7, p. 8). Bijection is the precise mathematical sense of "perfect pairing".
- f : N to N given by f(x) = 2x is one-one but not onto (no x with 2x = 1); f : R to R given by f(x) = 2x is one-one AND onto (NCERT §1.3, Examples 8 & 9, pp. 8–9).
- f : R to R, f(x) = x^2 is neither one-one (f(-1) = f(1)) nor onto (no x with x^2 = -2) (NCERT §1.3, Example 11, p. 9).
- f : N to N defined as f(x) = x + 1 if x is odd and f(x) = x - 1 if x is even is both one-one and onto (NCERT §1.3, Example 12, pp. 9–10).
- Characteristic property of a finite set X: every one-one function f : X to X is onto, and every onto function f : X to X is one-one; this fails for infinite sets (NCERT §1.3, Remark after Examples 13–14, p. 10). The pigeonhole principle underlies this.
- Composition of functions: if f : A to B and g : B to C, then gof : A to C is given by gof(x) = g(f(x)) for all x in A (NCERT §1.4, Definition 8, p. 12). Read "gof" as "g composed with f" or "g after f".
- Composition is NOT commutative in general: for f(x) = cos x and g(x) = 3x^2 on R, gof(x) = 3 cos^2 x while fog(x) = cos(3x^2), and 3cos^2 x != cos 3x^2 at x = 0 (NCERT §1.4, Example 16, p. 12).

- A function f : X to Y is invertible if there exists g : Y to X with gof = I_X and fog = I_Y; this g is unique and is written f^{-1} (NCERT §1.4, Definition 9, p. 12).
- f is invertible iff f is one-one AND onto (bijective) (NCERT §1.4, p. 12). This is the central characterization of invertibility.
- Worked inverse example: f : N to Y, f(x) = 4x + 3 with Y = {y in N : y = 4x + 3 for some x in N} is invertible with inverse g(y) = (y - 3) / 4 (NCERT §1.4, Example 17, pp. 12–13).
- Miscellaneous result: intersection of two equivalence relations is an equivalence relation (NCERT Miscellaneous Example 18, p. 13).
- The number of one-one functions from {1,2,3} to itself is 3! = 6 (NCERT Miscellaneous Example 22, p. 14).
- Number of equivalence relations on {1,2,3} containing (1,2) and (2,1) is two (NCERT Miscellaneous Example 24, p. 14).
- These ideas underpin the rest of Class XII algebra and calculus: invertibility is used in inverse-trig and exponential-log work; equivalence relations underlie modular arithmetic and number theory; bijections appear in counting problems and group theory.
- A practical CUET technique: when the relation R is given as a set of ordered pairs, check each property by going through the list mechanically. When R is given as a rule (e.g., R = {(a, b) : 3 | a – b}), check the general assertion using algebraic manipulation.
- Standard examples to memorise for fast MCQ work: identity function I_X is bijective; constant function on a multi-element domain is many-one and not onto; characteristic functions are typically not invertible.

2.2 Definitions to memorise

Term	Definition	Page
Relation in A	Subset of A × A	2
Empty relation	R = ∅	2
Universal relation	R = A × A	2
Reflexive	(a, a) ∈ R for all a	2
Symmetric	(a, b) ∈ R ⇒ (b, a) ∈ R	2
Transitive	(a, b), (b, c) ∈ R ⇒ (a, c) ∈ R	2
Equivalence relation	Reflexive + Symmetric + Transitive	3
Equivalence class [a]	{x ∈ X : x R a}	4
Partition	Disjoint subsets whose union is X	4
One-one (injective)	$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$	7

Term	Definition	Page
Many-one	Not one-one	7
Onto (surjective)	Range f = co-domain	7
Into	Range f ⊊ co-domain	8
Bijective	One-one and onto	8
Composition gof	gof(x) = g(f(x))	12
Identity function I_X	I_X(x) = x	12
Invertible function	∃ g with gof = I_X, fog = I_Y	12
Inverse f^{-1}	Unique such g	12
Bijection ⇔ invertible	Theorem of §1.4	12
Finite-set rule	One-one ⇔ onto for f : X → X	10
Pigeonhole principle	Underlies finite-set rule	10
Trivial relation	Empty or universal	2
Residue class mod n	Equivalence class under "n divides a – b"	4
Composition order	Apply right one first	12
Non-commutativity	gof ≠ fog in general	12

2.3 Diagrams / processes to remember

- **Fig 1.1 (p. 3):** illustrates the "perpendicular lines" relation — useful visual for showing why perpendicularity is symmetric but not transitive (L1 perp L2 and L2 perp L3 forces L1 parallel L3, not perpendicular).
- **Fig 1.2 (i)–(iv) (p. 8):** the four arrow-diagrams for f1, f2, f3, f4 showing all four combinations of one-one/many-one with onto/not-onto; the canonical visual for testing injectivity and surjectivity from a mapping diagram.
- **Fig 1.3 (p. 9):** graph of f(x) = 2x from R to R — straight line through the origin demonstrating bijectivity (horizontal-line test passes; line meets every horizontal level exactly once).
- **Fig 1.4 (p. 9):** illustration of f : N to N with the odd/even swap rule — depicts how every natural number is hit exactly once (bijection on N).
- **Fig 1.5 (p. 12):** composition diagram showing A —f→ B —g→ C and the combined arrow A —gof→ C; reinforces order ("g of f", apply f first then g).
- **Process — test reflexive/symmetric/transitive:** (i) for each a, check (a, a) ∈ R; (ii) for each (a, b) ∈ R, check (b, a) ∈ R; (iii) for each chain (a, b), (b, c) ∈ R, check (a, c) ∈ R. Equivalence iff all three hold.
- **Process — test injectivity:** assume $f(x_1) = f(x_2)$; derive $x_1 = x_2$. If derivation fails, find a counter-example.

- **Process — test surjectivity:** pick arbitrary y in co-domain; solve y = f(x) for x and check x lies in domain. If solvable for every y, function is onto.
- **Process — find inverse:** solve y = f(x) for x in terms of y; verify gof = I_X and fog = I_Y.
- **Process — compute composition:** identify inner and outer functions; substitute inner output as outer input.

2.4 Common confusions / NTA trap points

- Reversed composition: students write fog when gof is asked. Remember (gof)(x) = g(f(x)) — the function written **closer to x** acts first.
- Treating "onto" loosely: a function is onto only if Range = Co-domain. f : N to N, f(x) = 2x is NOT onto (1 has no pre-image); the same formula on f : R to R IS onto. The co-domain matters as much as the formula.
- Confusing reflexive with the universal relation: every element being related to itself is required, but other pairs may be absent. A reflexive relation need not be the universal relation.
- Symmetry vs antisymmetry: students mark "a <= b" as symmetric. It is reflexive and transitive but NOT symmetric (a <= b does not give b <= a).
- Forgetting the bijection condition for invertibility: a function with a "formula inverse" like sqrt is not automatically invertible unless domain and co-domain are restricted to make it bijective.
- For a finite set X, one-one and onto are equivalent for f : X to X — but this equivalence FAILS for infinite sets (e.g., f : N to N, f(x) = 2x is one-one but not onto).
- Confusing "many-one" with "many-many"; functions are never many-many.
- Mis-identifying the image of x^2 as all real numbers; image is [0, ∞), not R.
- Treating x → 1/x as a function on R; the natural domain is R \ {0}.
- Forgetting empty domain quirks; functions on the empty set are vacuously both one-one and onto.
- Confusing the inverse relation R^{-1} (a set-theoretic concept) with the inverse function f^{-1} (defined only when f is bijective).
- Mis-reading an arrow diagram: an arrow from x to y means f(x) = y; the function is many-to-one if two arrows arrive at the same y.
- Treating "identity function" as f(x) = 1; the identity is I_X(x) = x, not the constant 1.

2.5 Key formulas & theorems

Formula	Statement	NCERT page
Relation	$R \subseteq A \times A$	2
Empty relation	$R = \varnothing$	2

Formula	Statement	NCERT page
Universal relation	R = A × A	2
Reflexive condition	(a, a) ∈ R for all a	2
Symmetric condition	(a, b) ∈ R ⇒ (b, a) ∈ R	2
Transitive condition	(a, b), (b, c) ⇒ (a, c)	2
Equivalence relation	All three above	3
Equivalence class	[a] =	4
Partition by ≡	Classes are disjoint, cover X	4
Number of equivalence classes mod n	n	4
One-one	$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$	7
Onto	Range = Y	7
Bijective	One-one + onto	8
Composition	(gof)(x) = g(f(x))	12
Composition associativity	h ∘ (g ∘ f) = (h ∘ g) ∘ f	12
Identity	I_X(x) = x	12
Inverse condition	gof = I_X and fog = I_Y	12
Invertibility ⇔ bijection	Theorem	12
Finite-set self-map	one-one ⇔ onto	10
2x on N	one-one, not onto	8
2x on R	bijection	9
x^2 on R	neither	9
4x + 3 inverse	(y − 3)/4	13
Intersection of equivalence relations	Equivalence	13
Number of bijections {1,2,3} → {1,2,3}	3! = 6	14
Equivalence relations on {1,2,3} with (1,2),(2,1)	2	14

2.6 Solved examples (NCERT-grounded)

Example A (NCERT Example 5, p. 3). Show R = {(a, b) : 2 divides (a − b)} on Z is an equivalence relation.

Step 1 — reflexive: 2 | (a − a) = 0 ⇒ (a, a) ∈ R. Step 2 — symmetric: 2 | (a − b) ⇒ 2 | −(a − b) = (b − a). Step 3 — transitive: 2 | (a − b), 2 | (b − c) ⇒ 2 | (a − b) + (b − c) = (a − c). **Equivalence**.

Example B (NCERT Example 11, p. 9). Show $f(x) = x^2$ on R is neither one-one nor onto.

Step 1 — not one-one: $f(-1) = 1 = f(1)$ but $-1 \neq 1$. Step 2 — not onto: there is no $x \in R$ with $x^2 = -2$. Step 3 — conclude: **neither injective nor surjective**.

Example C (NCERT Example 12, pp. 9–10). Show $f : N \to N$ defined as $f(x) = x + 1$ (odd x) and $f(x) = x - 1$ (even x) is a bijection.

Step 1 — one-one: if $f(x_1) = f(x_2)$ with both odd or both even, immediate cancellation; mixed parity is impossible because outputs differ in parity. Step 2 — onto: every natural y is $f(y - 1)$ if y even and $y + 1$ odd, or $f(y + 1)$ if y odd and $y + 1$ even. Step 3 — conclude: **bijection on N**.

Example D (NCERT Example 16, p. 12). $f(x) = \cos x$, $g(x) = 3x^2$. Find gof and fog; check $(gof)(0) = (fog)(0)$.

Step 1 — gof: $g(f(x)) = g(\cos x) = 3 \cos^2 x$. Step 2 — fog: $f(g(x)) = f(3x^2) = \cos(3x^2)$. Step 3 — at $x = 0$: $gof(0) = 3 \cdot 1 = 3$; $fog(0) = \cos 0 = 1$. **Different**; composition not commutative.

Example E (NCERT Example 17, pp. 12–13). Find inverse of $f : N \to Y$, $f(x) = 4x + 3$, $Y = \{y \in N : y = 4x + 3, x \in N\}$.

Step 1 — solve for x: $y = 4x + 3 \Rightarrow x = (y - 3)/4$. Step 2 — verify gof: $g(f(x)) = ((4x + 3) - 3)/4 = x$. ✓ Step 3 — verify fog: $f(g(y)) = 4 \cdot (y - 3)/4 + 3 = y$. ✓ **Inverse:** $g(y) = (y - 3)/4$.

🎯 Practice MCQs

📊 PYQ Alignment

In CUET (UG) 2023–25 Mathematics papers, "Relations and Functions" reliably contributes around 8–10 MCQs per year, with the highest concentration on (i) classifying a given small relation as reflexive/symmetric/transitive or equivalence, (ii) testing one-one and onto for a stated f(x) on N, Z or R, and (iii) computing gof or finding f^{-1} for linear functions of the form f(x) = ax + b — all of which sit on the worked examples and exercises in this chapter.