FREE EDITION · NOTES + 3 SAMPLE MCQS

CUET · MATHEMATICS · CLASS XII · CODE 319

# Vector Algebra

CUET unit: Vector Algebra

By **UniDrill** · NCERT-grounded study material

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## 📌 Snapshot

- Distinguishes scalar quantities (magnitude only — length, mass, time, speed) from vector quantities (magnitude + direction — displacement, velocity, force) and develops the directed-line-segment model.
- Builds the algebra of vectors in three-dimensional space using the standard basis $\hat{i}, \hat{j}, \hat{k}$, with position vectors of points and the formula $|r| = \sqrt{(x^2 + y^2 + z^2)}$.
- Establishes operations — addition (triangle, parallelogram, polygon laws), scalar multiplication, section formula, scalar (dot) product, and vector (cross) product — with their properties.
- Connects the dot product to angle/projection/perpendicularity, and the cross product to area of triangles/parallelograms, parallelism, and right-handed orientation.
- CUET frequently tests numerical computation of dot/cross products, angles, projections, unit vectors, direction cosines, section-formula points, and areas — the work is computation-heavy and formula-driven.



## 📖 Detailed Notes

### 2.1 Core concepts

- A scalar has only magnitude (e.g. length, mass, time, speed, volume), while a vector has both magnitude and direction (e.g. displacement, velocity, force, momentum) (NCERT §10.1, p. 338).
- A directed line segment from initial point A to terminal point B is a vector denoted AB; its magnitude |AB| is the distance between A and B, and is never negative (NCERT §10.2, p. 339).
- For a point P(x, y, z), the position vector OP has magnitude $|OP| = \sqrt{(x^2 + y^2 + z^2)}$ with respect to origin O (NCERT §10.2 Position Vector, p. 339).
- The direction angles $\alpha$, $\beta$, $\gamma$ are the angles OP makes with the positive x-, y-, z-axes; $l = \cos\alpha,\ m = \cos\beta,\ n = \cos\gamma$ are the direction cosines, and $lr,\ mr,\ nr$ (proportional to l, m, n) are the direction ratios a, b, c (NCERT §10.2 Direction Cosines, p. 340).
- The identity $l^2 + m^2 + n^2 = 1$ always holds, but in general $a^2 + b^2 + c^2 \neq 1$ (NCERT §10.2 Note, p. 341).

- Types of vectors — Zero vector (coincident initial/terminal points, no definite direction); Unit vector (magnitude 1, denoted â); Coinitial vectors (same initial point); Collinear vectors (parallel to the same line); Equal vectors (same magnitude and direction); Negative of a vector (same magnitude, opposite direction) (NCERT §10.3, p. 341).
- Free vectors — any vector may be displaced parallel to itself without changing its magnitude or direction; throughout this topic we deal with free vectors only (NCERT §10.3 Remark, p. 341).
- Triangle law of vector addition — if a girl moves A → B → C, then AB + BC = AC (NCERT §10.4, p. 343).
- Parallelogram law — if two vectors are represented by two adjacent sides of a parallelogram, their sum is the diagonal through their common point; the two laws are equivalent (NCERT §10.4, p. 344).
- Properties of vector addition — commutative (a + b = b + a) and associative ((a + b) + c = a + (b + c)); zero vector is the additive identity (NCERT §10.4 Properties, pp. 344–346).
- Multiplication by scalar $\lambda$ — $\lambda$a is collinear to a, has magnitude $|\lambda||a|$, and the same direction as a if $\lambda > 0$ or opposite direction if $\lambda < 0$; when $\lambda = 1/|a|$, $\lambda$a gives the unit vector â = a/|a| (NCERT §10.5, pp. 346–347).
- Component form — any vector r with terminal point P(x, y, z) is $r = x\hat{i} + y\hat{j} + z\hat{k}$, where x, y, z are scalar components and $|r| = \sqrt{(x^2 + y^2 + z^2)}$ (NCERT §10.5.1, pp. 347–348).
- Two vectors $a = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $b = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are equal iff $a_1 = b_1$, $a_2 = b_2$, $a_3 = b_3$; they are collinear iff $b_1/a_1 = b_2/a_2 = b_3/a_3 = \lambda$ (NCERT §10.5.1, pp. 348–349).
- Vector joining two points — $P_1P_2 = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$ with magnitude $\sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2)}$ (NCERT §10.5.2, p. 351).
- Section formula (internal) — point R dividing PQ internally in ratio m:n has position vector $(mb + na)/(m + n)$; (external) — $(mb - na)/(m - n)$; midpoint = $(a + b)/2$ (NCERT §10.5.3, pp. 352–353).
- Scalar (dot) product — $a \cdot b = |a||b| \cos \theta$, where $\theta$ is the angle between a and b; result is a scalar (NCERT §10.6.1, p. 355).
- Dot-product observations — a · b = 0 iff a ⊥ b (for nonzero vectors); $a \cdot a = |a|^2$; commutative (a · b = b · a); distributive over addition; $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$ and $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$ (NCERT §10.6.1, pp. 356–357).
- Component form of dot product — $a \cdot b = a_1b_1 + a_2b_2 + a_3b_3$; angle $\cos \theta = (a \cdot b)/(|a||b|)$ (NCERT §10.6.1, p. 357).
- Projection of a vector — projection of a on a directed line with unit vector p̂ is $a \cdot \hat{p}$; projection of a on vector b is $(a \cdot b)/|b|$ (NCERT §10.6.2, pp. 357–358).

- Vector (cross) product — $a \times b = |a||b| \sin\theta\, \hat{n}$, where n̂ is a unit vector perpendicular to both a and b such that (a, b, n̂) forms a right-handed system; result is a vector (NCERT §10.6.3, p. 363).
- Cross-product observations — a × b = 0 iff a ∥ b (for nonzero vectors); not commutative — $a \times b = -b \times a$ (anti-commutative); distributive over addition; $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$; $\hat{i} \times \hat{j} = \hat{k},\ \hat{j} \times \hat{k} = \hat{i},\ \hat{k} \times \hat{i} = \hat{j}$ (NCERT §10.6.3, pp. 363–365).
- Area applications — area of triangle with adjacent sides a, b is $\frac{1}{2}|a \times b|$; area of parallelogram with adjacent sides a, b is $|a \times b|$ (NCERT §10.6.3 Observations 8–9, p. 365).
- Determinant form — $a \times b = |\ \hat{i}\ \hat{j}\ \hat{k}\ ;\ a_1\ a_2\ a_3\ ;\ b_1\ b_2\ b_3\ |$; angle via $\sin\theta = |a \times b|/(|a||b|)$ (NCERT §10.6.3, pp. 363, 366).

## 2.2 Definitions to memorise

| Term | Definition | Page |
|---|---|---|
| Scalar | Quantity with only magnitude (real number) | 338 |
| Vector | Quantity with both magnitude and direction | 339 |
| Position vector of P(x, y, z) | OP with $\lvert OP\rvert = \sqrt{x^2 + y^2 + z^2}$ | 339 |
| Direction cosines (l, m, n) | cos $\alpha$, cos $\beta$, cos $\gamma$ with x-, y-, z-axes; $l^2 + m^2 + n^2 = 1$ | 340–341 |
| Direction ratios (a, b, c) | Numbers proportional to direction cosines (lr, mr, nr) | 340 |
| Zero vector | Initial and terminal points coincide; magnitude 0; no definite direction | 341 |
| Unit vector | Vector with magnitude 1; for vector a, $\hat{a} = a/\lvert a\rvert$ | 341, 347 |
| Coinitial vectors | Two or more vectors with the same initial point | 341 |
| Collinear vectors | Two or more vectors parallel to the same line | 341 |
| Equal vectors | Same magnitude and same direction | 341 |
| Negative of a vector | Same magnitude, opposite direction | 341 |
| Free vector | Vector subject to parallel displacement without change | 341 |
| Triangle law | AB + BC = AC | 343 |
| Parallelogram law | Sum of two coinitial vectors = diagonal of parallelogram on those sides | 344 |
| Scalar (dot) product | $a \cdot b = \lvert a\rvert\lvert b\rvert \cos\theta$ (scalar) | 355 |
| Projection of a on b | $(a \cdot b)/\lvert b\rvert$ | 358 |
| Vector (cross) product | | 363 |

| Term | Definition | Page |
|---|---|---|
| | `a × b = |a||b| sin θ n̂` (vector); n̂ ⊥ both, right-handed | |
| Section formula (internal) | R = `(mb + na)/(m + n)` | 352 |
| Section formula (external) | R = `(mb − na)/(m − n)` | 353 |
| Area of triangle (vectors a, b) | `½|a × b|` | 365 |
| Area of parallelogram (sides a, b) | `|a × b|` | 365 |

## 2.3 Diagrams / processes to remember

- **Fig 10.1 (p. 339):** Directed line, directed line segment AB showing magnitude with arrow direction.
- **Fig 10.2 (p. 340):** Three-dimensional right-handed rectangular coordinate system showing position vectors of points A, B, C with respect to origin O.
- **Fig 10.3 (p. 340):** Direction angles $\alpha$, $\beta$, $\gamma$ of position vector OP with x-, y-, z-axes; right-angled triangles OAP, OBP, OCP used to derive l = x/r, m = y/r, n = z/r.
- **Fig 10.7–10.8 (pp. 343–344):** Triangle law and polygon-extension for vector addition; subtraction as addition of negative.
- **Fig 10.9–10.10 (pp. 344–345):** Parallelogram law of vector addition; proof of commutativity via parallelogram ABCD.
- **Fig 10.13–10.14 (pp. 347–348):** Unit vectors î, ĵ, k̂ along OX, OY, OZ; component-form derivation by dropping perpendicular foot $P_1$.
- **Fig 10.16–10.17 (pp. 352–353):** Internal and external section formula construction.
- **Fig 10.19 (p. 356):** Angle $\theta$ between two vectors for scalar product.
- **Fig 10.20 (p. 357):** Projection vector AC for the four cases $0 < \theta < 90°$, $90° < \theta < 180°$, $180° < \theta < 270°$, $270° < \theta < 360°$.
- **Fig 10.22–10.24 (pp. 363–364):** Right-hand rule for cross product; orientation of î, ĵ, k̂ in a right-handed system.
- **Fig 10.26–10.27 (p. 365):** Area of triangle and parallelogram in terms of |a × b|.

## 2.5 Key formulas & theorems

| Formula | Statement | NCERT page |
|---|---|---|
| Position vector | OP = xî + yĵ + zk̂ | 339 |
| Magnitude | | r |
| DCs | l = cos $\alpha$, m = cos $\beta$, n = cos $\gamma$ | 340 |

| Formula | Statement | NCERT page |
|---|---|---|
| DC identity | $l^2 + m^2 + n^2 = 1$ | 341 |
| Unit vector | â = a/ | a |
| Equal vectors | All components equal | 348 |
| Collinear | $b_1/a_1 = b_2/a_2 = b_3/a_3 = \lambda$ | 349 |
| Vector between two points | $(x_2-x_1)\hat{i} + (y_2-y_1)\hat{j} + (z_2-z_1)\hat{k}$ | 351 |
| Section formula (internal) | R = (mb + na)/(m + n) | 352 |
| Section formula (external) | R = (mb − na)/(m − n) | 353 |
| Midpoint | (a + b)/2 | 353 |
| Dot product | a · b = | a |
| Component dot | $a \cdot b = a_1b_1 + a_2b_2 + a_3b_3$ | 357 |
| Perpendicularity | a · b = 0 | 356 |
| a · a | | a |
| Projection a on b | (a · b)/ | b |
| Cross product | a × b = | a |
| Component cross | Determinant with $\hat{i}, \hat{j}, \hat{k}$ | 363 |
| Anti-commutative | a × b = − b × a | 364 |
| Parallel test | a × b = 0 | 363 |
| Basis dot products | $\hat{i}\cdot\hat{i} = 1, \hat{i}\cdot\hat{j} = 0$ etc. | 357 |
| Basis cross products | $\hat{i} \times \hat{j} = \hat{k}$ etc. | 364 |
| Area of parallelogram | | a × b |
| Area of triangle | ½ | a × b |
| Triangle law | AB + BC = AC | 343 |
| Parallelogram law | Sum = diagonal | 344 |

## 2.6 Solved examples (NCERT-grounded)

**Example A (NCERT Example 14, p. 359).** Angle between $a = \hat{i} + \hat{j} - \hat{k}$ and $b = \hat{i} - \hat{j} + \hat{k}$.

Step 1 — dot product: $1\cdot1 + 1\cdot(-1) + (-1)\cdot1 = -1$. Step 2 — magnitudes: $|a| = |b| = \sqrt{3}$. Step 3 — cosine: $\cos\theta = -1/3 \Rightarrow$ $\boldsymbol{\theta = \cos^{-1}(-1/3)}$.

**Example B (NCERT Example 24, p. 367).** Area of triangle with vertices A(1,1,1), B(1,2,3), C(2,3,1).

Step 1 — vectors: AB = (0,1,2); AC = (1,2,0). Step 2 — cross product: AB × AC = (1·0 − 2·2, 2·1 − 0·0, 0·2 − 1·1) = (−4, 2, −1); magnitude = √21. Step 3 — area: ½ · √21 = **(1/2)√21 sq units**.

**Example C (NCERT Exercise 10.3 Q4, p. 361).** Projection of a = î + 3ĵ + 7k̂ on b = 7î − ĵ + 8k̂.

Step 1 — dot product: 7 − 3 + 56 = 60. Step 2 — |b|: √(49 + 1 + 64) = √114. Step 3 — projection: **60/√114**.

**Example D (NCERT Exercise 10.2 Q14, p. 354).** Show î + ĵ + k̂ is equally inclined to axes.

Step 1 — magnitude: √3. Step 2 — DCs: (1/√3, 1/√3, 1/√3). Step 3 — conclude: DCs equal ⇒ direction angles equal ⇒ **equally inclined**.

**Example E (Internal section formula).** Find R dividing P (a = î + 2ĵ + 3k̂) and Q (b = 4î + 5ĵ + 6k̂) in ratio 2:1.

Step 1 — apply formula: R = (2·b + 1·a)/(2 + 1). Step 2 — numerator: 2(4î + 5ĵ + 6k̂) + (î + 2ĵ + 3k̂) = 9î + 12ĵ + 15k̂. Step 3 — divide by 3: R = **3î + 4ĵ + 5k̂**.

## 2.4 Common confusions / NTA trap points

- Students confuse direction cosines `(l, m, n)` with direction ratios `(a, b, c)`; remember `l² + m² + n² = 1` always, but `a² + b² + c² ≠ 1` in general (NCERT §10.2 Note, p. 341).
- "Two collinear vectors are always equal in magnitude" — FALSE. Collinearity needs only parallelism, not equal magnitude (NCERT Exercise 10.1 Q5, p. 342).
- Internal vs external section formula sign — internal uses `mb + na` over `m + n`, while external uses `mb − na` over `m − n`. NTA likes to swap these (NCERT §10.5.3, pp. 352–353).
- Dot product is commutative (a · b = b · a), but cross product is anti-commutative (a × b = − b × a). A frequent distractor reverses this (NCERT §10.6.1 Observation 7, p. 356; §10.6.3 Observation 6, p. 364).
- The converse of `a × b = 0 ⇒ a or b is zero` is NOT true — a × b can vanish when a is parallel to b without either being zero (NCERT Exercise 10.4 Q8, p. 368).
- Projection of a on b is `(a · b)/|b|`, NOT `(a · b)/|a|`. Students often pick the wrong denominator (NCERT §10.6.2, p. 358).
- For triangle area, the factor is `½|a × b|`, not `|a × b|` (which is for parallelogram) — easy to mix up (NCERT §10.6.3, p. 365).

# 🎯 Practice MCQs

# 📊 PYQ Alignment

Vector Algebra is among the highest-yielding Class XII Mathematics chapters in CUET — typically 8–10 MCQs per paper. Questions cluster around: computing dot/cross products and the angle between two vectors, projection of one vector on another, finding unit vectors and direction cosines, internal/external section formula problems, area of triangles/parallelograms using `|a × b|`, perpendicularity/parallelism conditions, and statement-based identification of types of vectors. Numerical-answer-style and matching questions appear frequently, so memorising the component-form formulas and the right-handed cross-product cycle $\hat{i} \to \hat{j} \to \hat{k}$ pays off.