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CUET · PHYSICS · CLASS XI · CODE 322

Gravitation

CUET unit: Gravitation

By **UniDrill** · NCERT-grounded study material

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📌 Snapshot

- Establishes Kepler's three empirical laws of planetary motion and shows how Newton's universal law of gravitation explains them, unifying terrestrial and celestial motion.
- Develops the inverse-square law $F = G m_1 m_2 / r^2$ with $G = 6.67 \times 10^{-11}$ N m²/kg² measured by Cavendish, plus shell theorems for extended bodies.
- Derives acceleration due to gravity $g = G M_E / R_E^2$, its decrease with altitude ($g(1 - 2h/R_E)$) and depth ($g(1 - d/R_E)$), with the curious result that g is maximum at the surface.
- Introduces gravitational potential energy $V = -G m_1 m_2 / r$, escape speed $v_e = \sqrt{2gR_E} = 11.2$ km/s, orbital speed $V = \sqrt{GM_E/(R_E+h)}$, satellite period T (Kepler's law applied to satellites), and total energy of an orbiting satellite $E = -GMm/(2a)$.
- CUET tests this unit heavily through direct formula recall, numerical substitution (escape velocity, orbital velocity, $T^2 \propto a^3$), and conceptual statement-based questions on shell theorems and conservation laws.

📖 Detailed Notes

2.1 Core concepts

Understanding gravitation began with the historical struggle to make sense of the apparent motions of the heavens (NCERT §7.1, p. 127–128). Ptolemy's second-century geocentric model put the Earth at the centre of a system of nested celestial spheres, with each planet riding on a small "epicycle" superposed on its larger deferent. The Indian astronomer Aryabhata (5th century AD) suggested a rotating Earth, but the heliocentric idea was definitively revived by Nicolaus Copernicus (1473–1543), who put the Sun at the centre and let the planets, including Earth, revolve around it in circles. Galileo's telescopic observations of Jupiter's moons and the phases of Venus provided the first direct empirical support for the Copernican picture.

The decisive quantitative breakthrough came from Johannes Kepler's analysis of Tycho Brahe's lifetime of naked-eye planetary observations. Working without a telescope, Tycho had compiled the most accurate position measurements of his era; Kepler, after years of arithmetic, distilled them into three remarkably clean laws (NCERT §7.2, p. 128–129):

1. Law of orbits. All planets move in elliptical orbits with the Sun situated at one of the two foci of the ellipse — not at the centre. A circle is the special case in which the two foci coincide.

2. Law of areas. The line joining a planet to the Sun sweeps out equal areas in equal intervals of time. A planet therefore moves faster when it is closer to the Sun (perihelion) and slower when farther away (aphelion).

3. Law of periods. The square of the orbital period T of a planet is proportional to the cube of the semi-major axis a of its elliptical orbit: $T^2 \propto a^3$. Numerically, $T^2/a^3 \approx 2.97 \times 10^{-19}\ s^2\ m^{-3}$ for all planets revolving around the Sun (NCERT Table 7.1, p. 129).

The law of areas has a deep dynamical origin: it is a direct consequence of **conservation of angular momentum** for a central force. The area swept per unit time is $dA/dt = L/(2m)$, and L is constant for any force directed along the line connecting two bodies (NCERT Eqs. 7.1–7.2, p. 129). Kepler's second law therefore tells us, in advance of Newton, that the Sun–planet force must be a central force.

Newton recognised that the same force keeping the Moon in orbit around the Earth is the very same that makes an apple fall — "all motion is universal". His famous Moon-test computation compared the centripetal acceleration of the Moon $a_m = 4\pi^2 R_m/T_m^2$ with the acceleration due to gravity g at Earth's surface, and found the ratio $g/a_m \approx 3600$, matching the ratio $(R_m/R_E)^2$ of the squares of the distances (NCERT §7.3, Eqs. 7.3–7.4, p. 129–130). This confirmed an inverse-square distance dependence.

Universal law of gravitation (NCERT §7.3, p. 130). Newton stated: every body in the universe attracts every other body with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. In vector form, the force on m_1 due to m_2 is

$\mathbf{F} = -(G m_1 m_2 / r^2)\,\hat{r}$ (NCERT Eq. 7.5),

attractive, along the line joining the two bodies. The proportionality constant G is a universal constant of nature.

For extended bodies the force is found by superposing the pairwise contributions of all the constituent mass elements. Two important results for spherically symmetric mass distributions ("shell theorems") follow from the calculation (NCERT §7.3, p. 131):

(1) A uniform spherical shell of matter attracts an external point mass as if all its mass were concentrated at its centre. (This is why we can treat Earth as a point mass when calculating g.)

(2) The gravitational force on a point mass placed inside a uniform spherical shell is **zero** everywhere inside — the contributions from different parts of the shell cancel exactly. This does NOT mean the shell shields the inside from external gravity; gravitational shielding is impossible.

Measurement of G (NCERT §7.4, p. 131–132). Henry Cavendish in 1798 made the first laboratory measurement using a torsion balance. Two large lead spheres (M ≈ 0.16

kg each in the standard textbook description) were brought near two small lead balls (m) suspended from the ends of a light horizontal rod held by a thin vertical fibre. Gravitational attraction twists the rod through an angle θ; calibrating the fibre's restoring torque $\tau\theta$ against the gravitational torque $(GMm/d^2)L$ gives G. The currently accepted value is

$G = 6.67 \times 10^{-11}$ N m²/kg² (NCERT Eq. 7.8, p. 132).

This is one of the most poorly known of the fundamental constants — gravity is the weakest of the four fundamental forces, and laboratory measurements are very delicate.

Acceleration due to gravity (NCERT §7.5, p. 132–133). For a body of mass m at Earth's surface, treating Earth as a uniform sphere of mass M_E and radius R_E,

$g = G M_E / R_E^2$ (NCERT Eq. 7.12, p. 133).

Knowing g, R_E and G, one can solve for $M_E \approx 6 \times 10^{24}$ kg. Cavendish, having measured G, was then able to "weigh the Earth" — a stunning achievement.

Variation of g with height and depth (NCERT §7.6, p. 133–134). At height h above the surface,

$g(h) = G M_E / (R_E + h)^2 \approx g(1 - 2h/R_E)$ for $h \ll R_E$ (Eq. 7.15).

At depth d below the surface (treating Earth as uniform density), only the smaller sphere of radius $(R_E - d)$ contributes — the outer shell exerts no force by the shell theorem — and

$g(d) = g(1 - d/R_E)$ (Eq. 7.19, p. 134).

Both formulas predict a decrease in g, so **g is maximum at Earth's surface**, falling off whether one moves up into space or down into a mine. This counter-intuitive result is a frequent CUET trap. At the centre of the Earth g = 0.

Gravitational potential energy (NCERT §7.7, p. 134–135). Near the surface the familiar W(h) = mgh is an approximation. The general expression, taking PE to be zero at infinity, is

$V(r) = -G m_1 m_2 / r$ (NCERT Summary point 5, p. 140).

The minus sign reflects the fact that gravity does positive work on a body falling toward another body — the system loses PE as r decreases. Only PE differences are physically meaningful; the choice of zero at infinity is conventional but standard. The **gravitational potential** at a point is the PE per unit mass placed at that point.

Escape speed (NCERT §7.8, p. 135–136). Imagine projecting a body of mass m radially outward from Earth's surface with initial speed v_i. By energy conservation, the body can escape to infinity (where its KE and PE are both zero) only if its total mechanical energy is non-negative:

$(1/2)\, m v_i^2 - G m M_E / R_E \geqslant 0$.

The minimum (escape) speed is therefore

$v_e = \sqrt{2 G M_E / R_E} = \sqrt{2 g R_E} \approx 11.2$ km/s.

Two important consequences: v_e is independent of the projected body's mass, and independent of the direction of projection (provided the trajectory does not re-enter the Earth). The Moon's escape speed is much smaller, about 2.3 km/s — one-fifth of Earth's — which is why the Moon could not retain any atmosphere over geological time.

Earth satellites (NCERT §7.9, p. 136–137). A satellite of mass m in a circular orbit of radius $r = R_E + h$ needs centripetal force GMm/r^2 supplied by gravity. Setting $mv^2/r = GMm/r^2$ gives the orbital speed

$v_{orb} = \sqrt{G M_E / (R_E + h)}$ (NCERT Eq. 7.36, p. 137).

For a satellite skimming Earth's surface ($h \to 0$), $v_{orb} = \sqrt{g R_E} \approx 7.9$ km/s. The relationship between escape and orbital speeds is $v_e = \sqrt{2}\, v_{orb}$ — escape needs exactly $\sqrt{2}$ (~1.41) times the surface orbital speed.

The orbital **period** is $T = 2\pi r/v_{orb}$, so

$T^2 = (4\pi^2/G M_E) \times (R_E + h)^3$ (Eq. 7.39, p. 137).

This is Kepler's third law applied to artificial satellites. For a "low-Earth-orbit" satellite ($h \approx 0$), $T_0 = 2\pi \sqrt{R_E/g} \approx 85$ minutes.

Energy of an orbiting satellite (NCERT §7.10, p. 138). For a circular orbit of radius r:

$KE = (1/2)\, m\, v_{orb}^2 = +G M m / (2r)$, $PE = -G M m / r$, **$E = KE + PE = -G M m / (2r)$**.

The total mechanical energy is negative, indicating a bound system; $|KE| = (1/2)|PE|$ (a form of the virial theorem). For elliptical orbits the total energy is still $-GMm/(2a)$ where a is the semi-major axis. Whenever a satellite descends to a lower orbit, its total energy becomes more negative but its KE actually increases — a counter-intuitive consequence of the negative PE term.

2.2 Definitions to memorise

Term	Definition	Page
Geocentric model	Earth-centred Ptolemaic system	127
Heliocentric model	Sun-centred Copernican system	128
Ellipse	Locus of points whose distance sum from two foci is constant	128
Semi-major axis (a)	Half the longest diameter of an ellipse	128
Perihelion / Aphelion	Closest / farthest point of a planet from the Sun	128
Kepler's first law (orbits)	All planets move in elliptical orbits with the Sun at one focus	128

Term	Definition	Page
Kepler's second law (areas)	Line from Sun to planet sweeps equal areas in equal times	128
Kepler's third law (periods)	$T^2 \propto a^3$	129
Central force	Force directed along the line joining source and particle; conserves angular momentum	129
Universal Law of Gravitation	$F = G m_1 m_2 / r^2$	130
Universal gravitational constant G	$G = 6.67 \times 10^{-11}$ N m²/kg²	132
Shell theorem (outside)	Uniform shell attracts external mass as if mass at centre	131
Shell theorem (inside)	Uniform shell exerts no force on internal point mass	131
Acceleration due to gravity (g)	$g = G M_E / R_E^2 \approx 9.8$ m/s² at surface	133
g at height h	$g(h) \approx g(1 - 2h/R_E)$	133
g at depth d	$g(d) = g(1 - d/R_E)$	134
Gravitational PE	$V = -G m_1 m_2 / r$ (PE zero at infinity)	135
Gravitational potential at a point	PE of a unit mass at that point	135
Escape speed	$v_e = \sqrt{2 G M_E / R_E} = \sqrt{2 g R_E} \approx 11.2$ km/s	136
Orbital speed (circular)	$v = \sqrt{G M_E / (R_E + h)}$; at surface $\sqrt{g R_E}$	137
Period of close satellite T_0	$2\pi \sqrt{R_E/g} \approx 85$ min	137
Total energy of circular orbit	$E = -G M m / (2r)$	138
Virial relation		KE
Geostationary satellite	Satellite with T = 24 h, h ≈ 36 000 km, equatorial orbit	137 (general)
Polar satellite	Low-altitude (~500–800 km) satellite passing over poles	137 (general)
Weightlessness in orbit	Apparent zero weight due to common free-fall of astronaut and craft	138

2.3 Diagrams / processes to remember

- **Fig. 7.1 (a)-(b) (p. 128):** Ellipse with two foci, semi-major axis a, perihelion P and aphelion A; classic string-and-pencil construction with the Sun at one focus.

- **Fig. 7.2 (p. 128):** Planet sweeping area ΔA in time Δt — visualises Kepler's second law and motivates $dA/dt = L/(2m)$.
- **Fig. 7.3 / 7.4 (p. 130):** Vector form of gravitational force; net force on m_1 by superposition of forces from m_2, m_3, m_4.
- **Fig. 7.5 (p. 131):** Three equal masses at vertices of an equilateral triangle with mass 2m at the centroid — example of symmetry-based cancellation (net force on 2m is zero).
- **Fig. 7.6 (p. 131):** Cavendish's torsion-balance schematic with large spheres S_1, S_2 attracting small masses on bar AB suspended by a fibre.
- **Fig. 7.7 (p. 132):** Mass m in a mine at depth d; only the inner sphere of radius (R_E – d) contributes to g.
- **Fig. 7.8 (a)-(b) (p. 133–134):** g vs altitude (decreases as (1 – 2h/R_E)) and g vs depth (decreases as (1 – d/R_E)).
- **Fig. 7.9 (p. 135):** Four masses at the corners of a square — gravitational PE example involving six pairs.
- **Fig. 7.10 (p. 136):** Two solid spheres of masses M and 4M separated by 6R with the gravitational-neutral point at r = 2R from M.

2.4 Common confusions / NTA trap points

- g vs G: G is the universal gravitational constant (6.67×10^{-11} N m^2/kg^2); g is acceleration due to gravity at a specific location (≈ 9.8 m/s^2 at Earth's surface). g depends on Earth's mass, not on the body's mass.
- g is **maximum at the surface**; it decreases both above (factor 1 – 2h/R_E) and below (factor 1 – d/R_E) — a frequent NTA trap.
- Escape speed does **not** depend on the mass of the projected body, nor on its direction of projection — only on the location.
- Shell theorem: gravitational force inside a uniform spherical shell is zero, but the shell does **not** shield bodies outside it from gravitational forces on bodies inside. Gravitational shielding is impossible.
- Weightlessness in an orbiting satellite is **not** because gravity is weak there — it is because both astronaut and craft are in free fall toward Earth.
- mgh is only an approximation valid near Earth's surface; the exact PE is $-G m_1 m_2 / r$.
- Kepler's second law follows from conservation of angular momentum, valid for any central force (not just gravity).
- Total energy of a bound orbit is **negative**, KE is positive, PE is negative, and |KE| = ½|PE|.
- v_e/v_orb = √2 — escape speed is exactly √2 times the surface orbital speed.
- For a satellite raising its orbit, the kinetic energy decreases even though the total energy increases (less negative) — counter-intuitive.

- Gravity acts equally on satellites and astronauts inside them, so the relative weight is zero (apparent weightlessness).
- The acceleration due to gravity inside a uniform Earth varies **linearly** with distance from the centre ($g \propto r$ for $r < R_E$), reaching zero at the centre.

2.5 Key formulas table

Symbol	Formula	Meaning	NCERT page
F (gravity)	$F = G m_1 m_2 / r^2$	Universal law of gravitation	130, Eq. 7.5
G	6.67×10^{-11} N m²/kg²	Universal constant	132, Eq. 7.8
dA/dt	$dA/dt = L/(2m) = \text{const}$	Kepler's 2nd law	129, Eq. 7.1
Kepler's 3rd law	$T^2 = (4\pi^2/GM)\, a^3$	Period vs semi-major axis	137
g (surface)	$g = G M_E / R_E^2$	Surface gravity	133, Eq. 7.12
g (height)	$g(h) \approx g(1 - 2h/R_E)$	$h \ll R_E$ approximation	133, Eq. 7.15
g (depth)	$g(d) = g(1 - d/R_E)$	Uniform-density Earth	134, Eq. 7.19
V (PE)	$V(r) = -G m_1 m_2/r$	PE zero at infinity	135
W (near surface)	$W = mgh + W_0$	Surface approximation	134, Eq. 7.21
v_e	$v_e = \sqrt{2 G M_E/R_E} = \sqrt{2 g R_E}$	Escape speed	136, Eq. 7.31
v_e (Earth)	11.2 km/s	Numerical value	136
v_e (Moon)	2.3 km/s	Numerical value	136
v_{orb}	$v = \sqrt{G M_E/(R_E + h)}$	Circular orbital speed	137, Eq. 7.36
v_{orb} (surface)	$v = \sqrt{g R_E} \approx 7.9$ km/s	$h \to 0$	137
v_e/v_{orb}	$\sqrt{2}$	Ratio at same radius	136–137
T (satellite)	$T^2 = (4\pi^2/GM)(R_E + h)^3$	Period of satellite	137, Eq. 7.39
T_0 (close)	$T_0 = 2\pi \sqrt{R_E/g} \approx 85$ min	Low-Earth orbit period	137
KE (orbit)	$KE = + G M m / (2r)$	Kinetic in circular orbit	138
PE (orbit)	$PE = - G M m / r$	Potential in orbit	138
E (orbit)	$E = -G M m/(2r)$	Total energy, circular orbit	138, Eq. 7.42
E (ellipse)	$E = -G M m/(2a)$	Total energy, semi-major axis a	140, Summary 7

🎯 Practice MCQs

📊 PYQ Alignment

Gravitation is a high-yield CUET Physics unit, contributing roughly 8–10 MCQs per year across CUET 2023–2025. Direct-formula numericals on escape velocity ($v_e = \sqrt{2gR_E}$), orbital velocity ($V = \sqrt{GM/r}$ or $\sqrt{gR_E}$ for close orbits), variation of g with height/depth, and Kepler's third law ($T^2 \propto a^3$) dominate; statement-based and assertion-reason questions on shell theorems, weightlessness in satellites, and conservation of angular momentum (Kepler's second law) appear regularly.

CUET 2023–25 — Actual PYQs from this chapter

No PYQs from this chapter appeared in CUET 2023, 2024 or 2025.