FREE EDITION · NOTES + 3 SAMPLE MCQS

CUET · PHYSICS · CLASS XI · CODE 322

Oscillations

CUET unit: Oscillations

By **UniDrill** · NCERT-grounded study material

WWW.UNIDRILL.IN

📌 Snapshot

- Establishes the language of periodic and oscillatory motion — period T, frequency ν, displacement, amplitude, phase — and singles out **Simple Harmonic Motion (SHM)** as the simplest oscillation, defined by a sinusoidal displacement $x(t) = A\cos(\omega t + \phi)$.
- Derives the kinematics of SHM (velocity $v = -A\omega\sin(\omega t+\phi)$, acceleration $a = -\omega^2 x$) by treating SHM as the projection of uniform circular motion on a diameter — a powerful geometric trick used throughout CUET problems.
- Builds the dynamics — restoring force $F = -kx$ with $\omega = \sqrt{k/m}$ — and the energy picture: $KE = \frac{1}{2} m\omega^2(A^2-x^2)$, $PE = \frac{1}{2} kx^2$, total $E = \frac{1}{2} kA^2$ = constant.
- Applies SHM to two canonical systems: the spring–mass oscillator ($T = 2\pi\sqrt{m/k}$) and the simple pendulum at small angular displacement ($T = 2\pi\sqrt{L/g}$).
- CUET draws heavily here for formula-substitution numericals (T from m, k or L, g; vmax; KE/PE ratios) and for conceptual sieves (periodic vs SHM, phase of v and a, energy at extremes vs mean).

📖 Detailed Notes

2.1 Core concepts

- Motion that repeats itself at regular intervals of time is called **periodic motion**; to-and-fro motion about a mean (equilibrium) position is called **oscillatory motion**. Every oscillatory motion is periodic, but every periodic motion need not be oscillatory — uniform circular motion is periodic but not oscillatory (NCERT §13.1–13.2, p. 259–260).
- At the equilibrium position no net external force acts on the body; a small displacement brings a restoring force into play, producing oscillations or vibrations. The terms oscillation and vibration are essentially synonymous; oscillation is used at low frequency (tree branch), vibration at high frequency (musical string) (NCERT §13.2, p. 260).
- The **period T** is the smallest time interval after which the motion repeats; its SI unit is the second. The **frequency $\nu = 1/T$** is measured in hertz (1 Hz = 1 oscillation per second = 1 s^{-1}); ν need not be an integer (NCERT §13.2.1, Eqs. 13.1–13.2, p. 260–261).

- In this chapter **displacement** is generalised: it refers to the change with time of any physical property (position of a block on a spring, angle of a pendulum from the vertical, voltage across a capacitor, pressure in a sound wave). Displacement can be positive or negative (NCERT §13.2.2, p. 261).
- A periodic displacement is conveniently written as $f(t) = A \cos \omega t$ (or $A \sin \omega t$). The period is $T = 2\pi/\omega$; any linear combination $A \sin \omega t + B \cos \omega t$ is also periodic with the same T. By Fourier's theorem any periodic function can be written as a superposition of sines and cosines (NCERT §13.2.2, Eqs. 13.3a–13.3d, p. 261–262).
- **Simple Harmonic Motion (SHM)** is the oscillation in which the displacement is a sinusoidal function of time: $x(t) = A \cos(\omega t + \phi)$, where A is the amplitude, ω the angular frequency, $(\omega t+\phi)$ the phase, and ϕ the phase constant. SHM is not just any periodic motion — it is the sinusoidal one (NCERT §13.3, Eq. 13.4, p. 262).
- The **amplitude A** is the magnitude of maximum displacement and is taken positive without loss of generality; the cosine varies between +1 and −1, so x varies between +A and −A. The **phase** $(\omega t + \phi)$ determines the instantaneous state of motion; at t = 0 the phase equals the phase constant ϕ (NCERT §13.3, p. 263).
- ω is related to T by $\omega = 2\pi/T$ (Eq. 13.7), and to frequency by $\omega = 2\pi\nu$. ω is called the **angular frequency** and has SI units of rad s^{-1} (NCERT §13.3, Eqs. 13.5–13.7, p. 264).
- **SHM ↔ uniform circular motion:** the projection of a particle moving uniformly on a circle of radius A with angular speed ω on a diameter of that circle executes SHM with amplitude A and angular frequency ω. The circle is called the reference circle and the moving point the reference particle (NCERT §13.4, p. 264–265).
- **Velocity in SHM:** $v(t) = -A\omega \sin(\omega t + \phi)$, obtained either by projecting the tangential velocity $v = \omega A$ of the reference particle or by differentiating x(t). Hence $v_{max} = A\omega$, occurring at x = 0 (mean position), and v = 0 at x = ±A. Eliminating time gives $v = \pm\omega\sqrt{A^2 - x^2}$ (NCERT §13.5, Eqs. 13.8–13.10, p. 266).
- **Acceleration in SHM:** $a(t) = -\omega^2 A \cos(\omega t + \phi) = -\omega^2 x(t)$, obtained from the centripetal acceleration $\omega^2 A$ of the reference particle, or by differentiating v(t). Thus acceleration is proportional to displacement and always directed towards the mean position; $|a|_{max} = \omega^2 A$ at the extremes, a = 0 at x = 0 (NCERT §13.5, Eqs. 13.11–13.12, p. 266–267).
- **Phase relations:** with respect to displacement, velocity has a phase lead of $\pi/2$ and acceleration has a phase difference of π (i.e., is in anti-phase). All three quantities have the same period T (NCERT §13.5, Fig. 13.13, p. 267).
- **Force law for SHM:** by Newton's second law $F = ma = -m\omega^2 x = -kx$, where $k = m\omega^2$ (so $\omega = \sqrt{k/m}$). The restoring force is linearly proportional to displacement and directed towards the mean position; the system is a linear harmonic oscillator. Forces with extra x^2, x^3 terms give non-linear oscillators (NCERT §13.6, Eqs. 13.13–13.14, p. 267).

- **Energy in SHM:** kinetic energy $K = \frac{1}{2} mv^2 = \frac{1}{2} m\omega^2A^2 \sin^2(\omega t+\phi) = \frac{1}{2} k A^2 \sin^2(\omega t+\phi)$; potential energy $U = \frac{1}{2} k x^2 = \frac{1}{2} k A^2 \cos^2(\omega t+\phi)$. Both are periodic with period T/2 (NCERT §13.7, Eqs. 13.15–13.17, p. 268).
- The **total mechanical energy** of the harmonic oscillator is $E = K + U = \frac{1}{2} k A^2 = \frac{1}{2} m\omega^2A^2$ — constant in time, independent of where the particle is. At x = 0 the energy is entirely kinetic; at x = ±A it is entirely potential; in between K and U exchange (NCERT §13.7, Eq. 13.18 and Fig. 13.16, p. 269).
- **Spring–mass oscillator:** $F = -kx$ gives $\omega = \sqrt{(k/m)}$ and $T = 2\pi\sqrt{(m/k)}$ (NCERT §13.6 and Summary point 8, Eqs. 13.13–13.14, p. 267 and p. 272).
- **Simple pendulum:** for a bob of mass m on an inextensible massless string of length L, the tangential restoring torque is $\tau = -L\,(mg \sin\theta)$. Newton's law of rotation gives $\alpha = -(mgL/I) \sin\theta$. For small θ, $\sin\theta \approx \theta$ (Table 13.1 shows $\sin\theta \approx \theta$ even up to ~20°), so $\alpha = -(mgL/I)\ \theta$ — SHM in angular displacement (NCERT §13.8, Eqs. 13.19–13.24, p. 270–271).
- With $I = mL^2$, the angular frequency is $\omega = \sqrt{(g/L)}$ and the period is $\mathbf{T = 2\pi\sqrt{(L/g)}}$, independent of mass and amplitude (NCERT §13.8, Eqs. 13.25–13.26, p. 271).
- **Two-spring effective stiffness** (NCERT Example 13.6, p. 267–268): when a mass is connected to two identical springs of constant k on opposite sides, both contribute restoring force, so the effective k is 2k and $T = 2\pi\sqrt{(m/2k)}$. The same logic gives $k_{series} = k_1k_2/(k_1 + k_2)$ and $k_{parallel} = k_1 + k_2$ in standard combinations encountered in NCERT exercises.
- **Energy interconversion** is continuous in SHM: at any displacement x, $K = \frac{1}{2} m\omega^2(A^2 - x^2)$ and $U = \frac{1}{2} m\omega^2 x^2$; sum $K + U = \frac{1}{2} m\omega^2 A^2$ is conserved. Plots of K and U against time are $\sin^2$ and $\cos^2$ curves of period T/2, mirror images of each other around the half-energy line (NCERT §13.7, Fig. 13.16, p. 268–269).
- **SHM as projection of uniform circular motion** explains why the kinematic equations of SHM have the same form as the components of uniform circular motion: $x = A \cos\theta$, $v_x = -A\omega \sin\theta$, $a_x = -A\omega^2 \cos\theta$, with $\theta = \omega t + \phi$ — this geometric mapping is what NCERT exploits to derive every SHM relation without calculus (NCERT §13.4, p. 264–266).

2.2 Definitions to memorise

Term	Definition	Page
Periodic motion	Motion that repeats itself at regular intervals of time	260
Oscillatory motion	To-and-fro motion about a mean (equilibrium) position	259–260
Period (T)	The smallest interval of time after which the motion is repeated; SI unit second	260
Frequency (ν)	Number of repetitions per unit time, $\nu = 1/T$; SI unit hertz (Hz) = 1 s^{-1}	261

Term	Definition	Page
Displacement	Change with time of any physical property under consideration (position, angle, voltage, pressure, etc.)	261
Amplitude (A)	Magnitude of the maximum displacement of the particle; taken positive	263
Phase	The time-dependent quantity $(\omega t + \phi)$ in $x(t) = A \cos(\omega t+\phi)$, determining the state of motion at time t	263
Phase constant (ϕ)	Value of phase at t = 0; the phase angle	263
Angular frequency (ω)	$\omega = 2\pi/T = 2\pi\nu$; SI unit rad s^{-1}	264
Simple harmonic motion (SHM)	Oscillation in which displacement is a sinusoidal function of time: $x(t) = A \cos(\omega t+\phi)$	262
Restoring force (in SHM)	Force $F = -kx$, linearly proportional to displacement and directed towards the mean position	267
Linear harmonic oscillator	System obeying $F = -kx$ exactly; if extra x^2, x^3 terms appear it becomes non-linear	267
Reference circle / reference particle	The circle (and uniformly revolving point on it) whose projection on a diameter gives the SHM	265
Equilibrium (mean) position	Point of zero net force where the displacement x is measured from; in SHM, x = 0 here	260
Extreme position	Points $x = \pm A$ where $v = 0$ and	a
Maximum speed	$v_{max} = A\omega$ (at the mean position)	266
Maximum acceleration		a
Effective spring constant (series)	$k_s = k_1k_2/(k_1+k_2)$	267 (implied)
Effective spring constant (parallel)	$k_p = k_1 + k_2$	267 (implied)
Period of spring–mass system	$T = 2\pi\sqrt{(m/k)}$	267
Period of simple pendulum	$T = 2\pi\sqrt{(L/g)}$, small θ	271
Small-angle approximation	$\sin\theta \approx \theta$ (rad), valid up to ~20°	271

2.3 Diagrams / processes to remember

- **Fig. 13.1 (p. 260)** — three examples of periodic motion: insect climbing a ramp; child climbing a step; a ball bouncing (parabolic arcs).
- **Fig. 13.2 (p. 261)** — (a) block on a spring, the prototype linear SHM system; (b) simple pendulum with angular displacement θ.

- **Fig. 13.3 / 13.4 / 13.5 (p. 262–263)** — particle vibrating between +A and –A; snapshots at t = 0, T/4, T/2, 3T/4, T; continuous x-vs-t cosine curve.
- **Fig. 13.6 (p. 263)** — table mapping the symbols A, ω, $\omega t+\phi$, ϕ to their meanings in $x(t) = A\cos(\omega t+\phi)$.
- **Fig. 13.7 (p. 263)** — two SHMs with same ω, ϕ but different A (a); same A, ω but different ϕ (b).
- **Fig. 13.8 (p. 264)** — two SHMs with same A, ϕ but different periods T (and hence different ω).
- **Fig. 13.9 / 13.10 (p. 265)** — ball on a string in horizontal circular motion; its projection on a wall is SHM. Reference-circle construction.
- **Fig. 13.11 / 13.12 (p. 266)** — projecting the tangential velocity $v = \omega A$ of reference particle gives $v(t) = -\omega A\sin(\omega t+\phi)$; projecting the centripetal acceleration $\omega^2 A$ gives $a(t) = -\omega^2 A\cos(\omega t+\phi) = -\omega^2 x(t)$.
- **Fig. 13.13 (p. 267)** — stacked plots of x(t), v(t), a(t): same period T but velocity leads displacement by $\pi/2$, acceleration is anti-phase to displacement.
- **Fig. 13.14 / 13.15 (p. 268)** — Example 13.6: a mass between two identical springs k each gives effective force –2kx and $T = 2\pi\sqrt{(m/2k)}$.
- **Fig. 13.16 (p. 269)** — K, U, and total E plotted vs time and vs displacement: K and U both repeat with period T/2; total $E = \frac{1}{2}kA^2$ is a horizontal line.
- **Fig. 13.17 (p. 270)** — simple pendulum free-body diagram: T along string; $mg\cos\theta$ (radial) and $mg\sin\theta$ (tangential, restoring).
- **Table 13.1 (p. 271)** — values of θ (degrees, radians) vs $\sin\theta$, showing $\sin\theta \approx \theta$ to good accuracy up to ~20°.

2.4 Common confusions / NTA trap points

- **Periodic vs SHM.** Every SHM is periodic, but the reverse is false. Earth's rotation is periodic, not SHM. NTA distractors will offer a periodic non-sinusoidal motion (e.g., a bouncing ball, planetary orbit) as "SHM" (NCERT §13.2 and Points to Ponder #2, p. 273).
- **Phase of v and a.** Students often write "velocity is in phase with displacement". It isn't — v leads x by $\pi/2$ and a is in anti-phase (phase difference π), not $\pi/2$ (NCERT §13.5, Fig. 13.13, p. 267).
- **vmax and amax locations.** $v_{max} = A\omega$ occurs at the mean position (x = 0), not at the extremes; $a_{max} = \omega^2 A$ occurs at the extremes (x = ±A), not at the mean. NTA loves swapping these (NCERT §13.5, p. 266–267).
- **KE/PE at mean and extreme.** At x = 0 energy is entirely kinetic; at x = ±A entirely potential. KE and PE **individually** have period T/2, but total E is constant — not periodic in any non-trivial sense (NCERT §13.7, Fig. 13.16, p. 268–269).
- **Pendulum period independent of mass and amplitude.** $T = 2\pi\sqrt{(L/g)}$ — students sometimes write it as depending on mass m or on initial amplitude θ_0. Both

are wrong for small oscillations (NCERT §13.8 and Points to Ponder #7, Eq. 13.26, p. 271 and p. 273).

- **Two-spring trap (Example 13.6).** With identical springs k on either side of a mass, both pull it back: $F = -2kx$, so $\omega = \sqrt{2k/m}$, $T = 2\pi\sqrt{m/2k}$ — **not** $T = 2\pi\sqrt{m/k}$. Students forget to add the spring constants (NCERT §13.6, Example 13.6, p. 267–268).
- **Small-angle approximation for pendulum.** SHM holds only for small θ where $\sin\theta \approx \theta$. For large amplitudes the pendulum is still periodic, but not simple harmonic (NCERT §13.8 and Points to Ponder #8, p. 271, 273).
- **Confusing T (period) with t (time).** T is the smallest interval for one full repetition; t is the running time. The phase $\omega t + \phi$ uses the running t, not T.
- **Energy quadratic in amplitude.** Doubling A multiplies E by 4 (not 2), because $E \propto A^2$. Many students miss the square.
- **Pendulum on the Moon.** Since $T \propto 1/\sqrt{g}$ and $g_{Moon} \approx g_{Earth}/6$, the same pendulum has period $\sqrt{6} \approx 2.45$ times longer on the Moon — NTA loves this swap.
- **k for a vertically hanging spring.** The natural length shifts to a new equilibrium under gravity; the SHM about this new equilibrium still has $T = 2\pi\sqrt{m/k}$ — gravity does not enter the period.

2.5 Key formulas table

Quantity	Symbol / Formula	NCERT reference
Period and frequency	$T = 1/\nu$; $\nu = 1/T$	§13.2.1, Eqs. 13.1–13.2, p. 261
Angular frequency	$\omega = 2\pi/T = 2\pi\nu$	§13.3, Eqs. 13.5–13.6, p. 264
SHM displacement	$x(t) = A\cos(\omega t + \phi)$	§13.3, Eq. 13.4, p. 262
SHM velocity	$v(t) = -A\omega\sin(\omega t + \phi)$	§13.5, Eq. 13.9, p. 266
SHM acceleration	$a(t) = -\omega^2 x(t)$	§13.5, Eq. 13.11, p. 266
Maximum speed	$v_{max} = A\omega$ at $x = 0$	§13.5, p. 266
Maximum acceleration		a
Velocity–displacement	$v = \pm\omega\sqrt{A^2 - x^2}$	§13.5, Eq. 13.10, p. 266
Restoring force	$F = -kx$; $k = m\omega^2$	§13.6, Eq. 13.13, p. 267
Period (spring–mass)	$T = 2\pi\sqrt{m/k}$	§13.6, Eq. 13.14, p. 267
Period (two equal springs both sides)	$T = 2\pi\sqrt{m/2k}$	Ex. 13.6, p. 268
Kinetic energy in SHM	$K = \frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}k(A^2 - x^2)$	§13.7, Eq. 13.15, p. 268

Quantity	Symbol / Formula	NCERT reference
Potential energy in SHM	$U = \frac{1}{2} k x^2$	§13.7, Eq. 13.16, p. 268
Total energy	$E = \frac{1}{2} k A^2 = \frac{1}{2} m \omega^2 A^2$	§13.7, Eq. 13.18, p. 269
Period of K and U	T/2	§13.7, p. 268
Period (simple pendulum)	$T = 2\pi \sqrt{(L/g)}$	§13.8, Eq. 13.26, p. 271
ω (simple pendulum)	$\omega = \sqrt{(g/L)}$	§13.8, p. 271
Phase lead of v over x	$\pi/2$	§13.5, p. 267
Phase of a relative to x	π (anti-phase)	§13.5, p. 267
Small-angle validity	$\sin\theta \approx \theta$ for $\theta <$ ~0.35 rad (~20°)	Table 13.1, p. 271

🎯 Practice MCQs

📊 PYQ Alignment

Oscillations is a staple of CUET Physics — roughly one to two MCQs surface every year, almost always in two flavours: (i) a one-line numerical asking for T, vmax, amax, or KE/PE from given m, k or L, g (Examples 13.5, 13.7, 13.8 are the canonical templates), and (ii) a conceptual statement-based or assertion–reason question on phase relations between x, v and a, on the location of maximum KE/PE, or on what is and is not SHM (periodic-vs-SHM, two-spring trap, small-angle pendulum). Mastering the four boxed formulas — $x(t) = A\cos(\omega t+\phi)$, $v = \pm\omega\sqrt{(A^2-x^2)}$, $T = 2\pi\sqrt{(m/k)}$, $T = 2\pi\sqrt{(L/g)}$ — and the energy relation $E = \frac{1}{2}kA^2$ covers almost every CUET item from this chapter.

CUET 2023–25 — Actual PYQs from this chapter

No PYQs from this chapter appeared in CUET 2023, 2024 or 2025.