

CUET · PHYSICS · CLASS XI · CODE 322

System of Particles and Rotational Motion

CUET unit: System of Particles and Rotational Motion

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Snapshot

- Bridges the gap between point-particle mechanics and extended-body dynamics by treating a rigid body as a system of particles whose inter-particle distances are fixed.
- Develops the concept of **centre of mass (CM)** for two, three and n-particle systems and for homogeneous bodies, and shows that the CM moves as if all external forces acted on a single particle of total mass M (Eq. 6.11).
- Introduces the **vector (cross) product** and uses it to define **torque** $\tau = r \times F$, **angular momentum** $L = r \times p$, and the linear–angular velocity relation $v = \omega \times r$.
- Establishes the rotational analogues of Newton's laws: $\tau = dL/dt$, $\tau = I\alpha$, $K = \frac{1}{2}I\omega^2$, $L = I\omega$, with conservation of angular momentum when $\tau_{\text{ext}} = 0$.
- Defines **moment of inertia** $I = \sum m_i r_i^2$ and **radius of gyration** k ($I = Mk^2$), and tabulates I for ring, rod, disc, cylinder and solid sphere — the basis of every numerical in this chapter.

Detailed Notes

2.1 Core concepts

- A **rigid body** is one in which distances between all pairs of particles remain unchanged; a rigid body fixed at a point or along a line has only rotational motion, otherwise its motion is pure translation or translation + rotation (NCERT §6.1, p. 92–95).
- In pure translation every particle has the same velocity at any instant; in **rotation about a fixed axis** every particle moves in a circle whose plane is perpendicular to the axis and whose centre lies on the axis (NCERT §6.1.1, p. 93–95).
- **Centre of mass of two particles** on a line: $X = (m_1x_1 + m_2x_2)/(m_1 + m_2)$; for equal masses the CM is midway between them (NCERT §6.2 Eq. 6.1, p. 95–96).
- **CM of n particles**: $X = \sum m_i x_i / M$, $Y = \sum m_i y_i / M$, $Z = \sum m_i z_i / M$; in vector form $R = \sum m_i r_i / M$ (NCERT §6.2 Eqs. 6.4a–d, p. 96).
- For three equal masses at the vertices of a triangle, the CM coincides with the **centroid** (NCERT §6.2, p. 96).

- For a continuous body the sums become integrals: $R = (1/M) \int r \, dm$; if the origin is chosen at the CM, $\int r \, dm = 0$ (NCERT §6.2 Eqs. 6.5a–b, 6.6, p. 97).
- By symmetry, the CM of a homogeneous rod, ring, disc or sphere coincides with its **geometric centre** (NCERT §6.2, p. 97).
- **Motion of the CM:** $MA = F_{\text{ext}}$ — the CM moves as if all mass were concentrated there and all external forces acted on it; internal forces (Newton's third-law pairs) sum to zero (NCERT §6.3 Eqs. 6.10–6.11, p. 99). Illustrated by a projectile that explodes mid-air: the CM continues along the original parabolic path (Fig. 6.12, p. 100).
- **Linear momentum of a system:** $P = \sum m_i v_i = MV$; $dP/dt = F_{\text{ext}}$; if $F_{\text{ext}} = 0$ then $P = \text{constant}$ (law of conservation of total linear momentum) (NCERT §6.4 Eqs. 6.14–6.18, p. 100–101). Examples: binary stars and radioactive decay of moving radium (Figs. 6.13–6.14, p. 101).
- **Vector product:** $c = a \times b$ with $|c| = ab \sin \theta$, c perpendicular to the plane of a and b , direction given by the right-hand screw / right-hand rule; $a \times b = -b \times a$; distributive over addition; $i \times j = k$, $j \times k = i$, $k \times i = j$ (NCERT §6.5, p. 102–103).
- **Angular velocity** $\omega = d\theta/dt$ is a vector along the axis of rotation; for any particle of a rotating rigid body $v_i = \omega r_i$, and in vector form $v = \omega \times r$ (NCERT §6.6 Eqs. 6.19–6.20, p. 103–105).
- **Angular acceleration** $\alpha = d\omega/dt$; for a fixed axis the vector equation reduces to the scalar $\alpha = d\omega/dt$ (NCERT §6.6.1 Eqs. 6.21–6.22, p. 105).
- **Torque** (moment of force) $\tau = r \times F$ with magnitude $\tau = rF \sin \theta = r_{\perp} F = rF_{\perp}$; SI unit N m; dimensions ML^2T^{-2} (same as energy but a vector) (NCERT §6.7.1 Eqs. 6.23–6.24, p. 106).
- **Angular momentum** of a particle $l = r \times p$; magnitude $l = rp \sin \theta = r_{\perp} p$; and $dl/dt = \tau$ — the rotational analogue of $F = dp/dt$ (NCERT §6.7.2 Eqs. 6.25a, 6.26, 6.27, p. 106–107).
- For a system of particles $L = \sum l_i$ and $dL/dt = \tau_{\text{ext}}$ (internal torques cancel in pairs by Newton's third law, assuming central forces) (NCERT §6.7 Eqs. 6.28a–b, p. 107–108).
- **Conservation of angular momentum:** if $\tau_{\text{ext}} = 0$, $L = \text{constant}$ (Eq. 6.29a); applied component-wise as L_x , L_y , L_z each constant (NCERT §6.7 Eq. 6.29b, p. 108).
- **Mechanical equilibrium of a rigid body** requires both translational equilibrium ($\sum F_i = 0$, Eq. 6.30a) and rotational equilibrium ($\sum \tau_i = 0$, Eq. 6.30b); coplanar forces reduce these to three scalar conditions (NCERT §6.8, p. 109–110).
- A **couple** is a pair of equal and opposite forces with different lines of action; net force zero, net torque nonzero — produces rotation without translation (NCERT §6.8, p. 110–111).

- **Principle of moments** for a lever: $d_1F_1 = d_2F_2$; mechanical advantage M.A. = $F_1/F_2 = d_2/d_1$ (NCERT §6.8.1 Eqs. 6.32a–b, p. 111).
- **Centre of gravity** is the point where the total gravitational torque vanishes; in uniform gravity the CG coincides with the CM (NCERT §6.8.2 Eq. 6.33, p. 112–113).
- **Moment of inertia** $I = \sum m_i r_i^2$ is the rotational analogue of mass; rotational kinetic energy $K = \frac{1}{2}I\omega^2$ (NCERT §6.9 Eqs. 6.34–6.35, p. 114). For a thin ring of radius R about its central axis, $I = MR^2$; for a pair of point masses $M/2$ at $\pm l/2$ on a light rod, $I = MI^2/4$ (NCERT §6.9, p. 115).
- **Radius of gyration** k is defined by $I = Mk^2$; e.g. for a rod about its midpoint $k^2 = L^2/12$, so $k = L/\sqrt{12}$; for a disc about its diameter $k = R/2$ (NCERT §6.9, p. 115). Dimensions of I are ML^2 ; SI unit $kg\ m^2$.
- **Standard moments of inertia (Table 6.1, p. 116)**: thin ring about its central axis = MR^2 , ring about a diameter = $MR^2/2$, thin rod about perpendicular axis at midpoint = $ML^2/12$, disc about its central axis = $MR^2/2$, disc about a diameter = $MR^2/4$, hollow cylinder about its axis = MR^2 , solid cylinder about its axis = $MR^2/2$, solid sphere about a diameter = $2MR^2/5$.
- **Kinematics of rotation about a fixed axis** (analogous to linear motion with constant acceleration): $\omega = \omega_0 + \alpha t$, $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$, $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ (NCERT §6.10 Eqs. 6.36–6.38, p. 117).
- **Dynamics**: work $dW = \tau d\theta$, power $P = \tau \omega$, and $\tau = I\alpha$ — Newton's second law for fixed-axis rotation (NCERT §6.11 Eqs. 6.39–6.41, p. 119–120).
- For rotation about a fixed axis $L_z = I\omega$; for bodies symmetric about the axis $L = L_z = I\omega$ (NCERT §6.12 Eqs. 6.42b, 6.42d, p. 121). If I is time-independent then $\tau = I\alpha$.
- **Conservation of angular momentum about a fixed axis**: if $\tau_{ext} = 0$, $I\omega = \text{constant}$ — illustrated by the spinning skater / swivel-chair experiment (folding arms decreases I , increases ω) and circus acrobats and pirouetting dancers (NCERT §6.12.1 Eq. 6.44, p. 122–123).
- **Composite-body trick for CM**: when a body is made of simpler shapes whose individual CMs are known (e.g. an L-lamina made of three squares), the CM of the composite is the mass-weighted average of the individual CMs — $R = (\sum m_i R_i)/(\sum m_i)$; illustrated in NCERT Example 6.2 (p. 98).
- **Why the CM motion is so useful**: because internal forces cancel out in pairs (Newton's third law), the equation $MA = F_{ext}$ for the CM is valid regardless of how complicated the internal motion is; this is what lets us describe the trajectory of an exploding shell, a swinging chain, or a rotating-and-translating wheel as a single particle of total mass M (NCERT §6.3, p. 99).
- **Scalar vs vector product distinction**: $a \cdot b = ab \cos \theta$ is a number; $a \times b = ab \sin \theta \hat{n}$ is a vector. The scalar product is commutative; the vector product is anti-commutative. Both are distributive over addition (NCERT §6.5, p. 102).

- **Angular vs ordinary frequency in rotation:** for uniform rotation, the magnitude of ω equals $2\pi \times$ revolution-frequency. When NCERT specifies ω in rad s^{-1} but a problem gives rpm, students must convert: ω (rad s^{-1}) = $2\pi \times (\text{rpm}/60)$ — exactly the conversion used in Example 6.11 (p. 117).

2.2 Definitions to memorise

| Term | Definition | Page |
|----------------------------------|---|-------------------|
| Rigid body | A body in which distances between all pairs of particles remain unchanged | 92 |
| Centre of mass (n particles) | $R = \sum m_i r_i / M$, with $M = \sum m_i$ | 96 |
| Vector product $a \times b$ | Vector of magnitude $ab \sin \theta$ perpendicular to the plane of a and b , direction by right-hand rule | 102 |
| Angular velocity ω | $d\theta/dt$; vector along the axis of rotation; $v = \omega \times r$ | 104 |
| Angular acceleration α | $d\omega/dt$ | 105 |
| Torque τ | $r \times F$; magnitude $rF \sin \theta$; SI unit N m | 106 |
| Angular momentum l | $r \times p$; magnitude $rp \sin \theta$ | 106 |
| Moment of inertia I | $\sum m_i r_i^2$ for a fixed axis | 114 |
| Radius of gyration k | Distance from axis defined by $I = Mk^2$ | 115 |
| Centre of gravity | Point where the total gravitational torque on the body is zero | 112 |
| Couple | Pair of equal and opposite forces with different lines of action — produces pure rotation | 110 |
| Mechanical advantage | $F_1/F_2 = d_2/d_1$ for a lever | 111 |
| Mechanical equilibrium | $\sum F_i = 0$ and $\sum \tau_i = 0$ | 109 |
| Linear momentum of system | $P = \sum m_i v_i = MV$ | 100 |
| Translation | Motion in which every particle has the same velocity at every instant | 93 |
| Rotation about a fixed axis | Motion in which every particle moves in a circle whose plane is perpendicular to the axis | 93 |
| Pure rolling | Combined motion of translation and rotation with no slip at the contact ($v_{CM} = \omega R$) | derived from §6.6 |
| Centroid | Geometric centre of a triangle / lamina; coincides with the CM only for equal masses or uniform density | 96 |
| Conservation of angular momentum | If $\tau_{ext} = 0$, $L = \text{constant}$; about a fixed axis $l\omega = \text{constant}$ | 108, 122 |

| Term | Definition | Page |
|---------------------------|--|------|
| Rotational kinetic energy | $K = \frac{1}{2} I \omega^2$ | 114 |
| Principle of moments | $d_1 F_1 = d_2 F_2$ for a lever in rotational equilibrium | 111 |
| Right-hand rule | Convention to fix the direction of a vector product (curl fingers from a to b, thumb points along $a \times b$) | 102 |

2.3 Diagrams / processes to remember

- Fig. 6.4 (p. 94): Rigid body rotating about the z-axis; each particle P_i describes a circle of radius r_i in a plane perpendicular to the axis with centre C_i on the axis.
- Fig. 6.8 (p. 97): Symmetry argument for the CM of a thin uniform rod — element at $+x$ is balanced by an equal element at $-x$, so $\int x \, dm = 0$.
- Fig. 6.11 (p. 98): CM of an L-shaped lamina decomposed into three unit squares — the worked example for treating composite bodies as a sum of CMs.
- Fig. 6.12 (p. 100): Exploding projectile — CM continues along the original parabolic path because the explosion is an internal force.
- Fig. 6.15 (p. 102): Right-hand-screw and right-hand rules for the direction of $a \times b$.
- Fig. 6.17(b) (p. 104): ω directed along the axis, $v = \omega \times r$ tangential to the circle.
- Fig. 6.18 (p. 106): $\tau = r \times F$ perpendicular to the r-F plane, direction by right-hand rule.
- Fig. 6.23 (p. 111): Lever with fulcrum, load arm d_1 , effort arm d_2 — pictorial form of the principle of moments.
- Fig. 6.25 (p. 112): Locating the CG of an irregular lamina by suspending it from two points and finding the intersection of plumb lines.
- Fig. 6.32 (p. 122): Spinning girl on a swivel chair / acrobat — visual of $I\omega = \text{constant}$.
- Table 6.1 (p. 116): Moments of inertia of standard shapes — this is the single most fact-checked table for CUET.
- Table 6.2 (p. 119): Side-by-side comparison of translational and rotational quantities — every analogy in this chapter is captured here.

2.4 Common confusions / NTA trap points

- **CM \neq centroid in general.** Equal masses at the vertices of a triangle give CM = centroid; unequal masses (Example 6.1) do not (p. 97–98). NTA loves to plant "centroid" as a distractor.
- **CG vs CM.** They coincide only in uniform gravity; the CM depends only on mass distribution, the CG also on g (p. 112–113).
- **Direction of τ and L.** Both are perpendicular to the plane of (r, F) or (r, p) respectively — students often guess "along r " or "along F ". Use right-hand rule.

- $\mathbf{a} \times \mathbf{b} \neq \mathbf{b} \times \mathbf{a}$. Vector product is anti-commutative ($\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$). Scalar product is commutative; do not mix them up (p. 102).
- **Table 6.1 axes.** Disc about its central axis is $MR^2/2$ but about a diameter is $MR^2/4$; ring about its central axis is MR^2 but about a diameter is $MR^2/2$ (p. 116). The axis matters.
- $\boldsymbol{\tau}_{\text{ext}} = \mathbf{0} \Rightarrow \mathbf{L}$ conserved, not $\boldsymbol{\omega}$ conserved. When a skater folds her arms, I decreases and ω increases so that $I\omega$ stays constant — many students wrongly think ω is conserved (p. 122–123).
- **Couple.** $\Sigma \mathbf{F} = \mathbf{0}$ does not imply $\Sigma \boldsymbol{\tau} = \mathbf{0}$; a couple has zero net force but nonzero net torque (p. 110, Points to Ponder 5).
- $\mathbf{K}_{\text{rot}} = \frac{1}{2}I\boldsymbol{\omega}^2$ uses $\boldsymbol{\omega}$ in rad s^{-1} . If you forget to convert $\text{rpm} \rightarrow \text{rad s}^{-1}$ you will be off by a factor of $(2\pi/60)^2 \approx 0.011$ — a classic numerical trap.
- **CM of L-shape composite.** Treat each rectangular sub-piece as a point mass located at its own CM; the overall CM is the mass-weighted average (NCERT Ex. 6.2). Wrongly taking the geometric centre of the whole L is a common error.
- $\boldsymbol{\tau} = I\boldsymbol{\alpha}$ applies for fixed axis only. If the axis is itself accelerating or non-principal, the relation needs the general $\boldsymbol{\tau} = d\mathbf{L}/dt$. Most NCERT problems are fixed-axis and the simpler form suffices.
- **Angular momentum $\mathbf{L} = \mathbf{r} \times \mathbf{p}$; magnitude $r p \sin \theta$.** A common slip is to write $|\mathbf{L}| = rp$ regardless of angle — wrong when the velocity is along (parallel to) \mathbf{r} , in which case $\mathbf{L} = \mathbf{0}$.

2.5 Key formulas table

| Quantity | Symbol / Formula | NCERT reference |
|------------------------------|--|---------------------------------------|
| CM of n particles (vector) | $\mathbf{R} = (\Sigma m_i \mathbf{r}_i)/M$ | §6.2, Eq. 6.4d, p. 96 |
| CM of continuous body | $\mathbf{R} = (1/M) \int \mathbf{r} dm$ | §6.2, Eq. 6.5b, p. 97 |
| Motion of CM | $M\mathbf{A} = \mathbf{F}_{\text{ext}}$ | §6.3, Eq. 6.11, p. 99 |
| Total linear momentum | $\mathbf{P} = M\mathbf{V}$; $d\mathbf{P}/dt = \mathbf{F}_{\text{ext}}$ | §6.4, Eqs. 6.14–6.17, p. 100 |
| Vector product magnitude | | $a \times b$ |
| Linear–angular velocity | $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$ | §6.6, Eq. 6.20, p. 105 |
| Torque (definition) | $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$; | $\boldsymbol{\tau}$ |
| Angular momentum (particle) | $\mathbf{l} = \mathbf{r} \times \mathbf{p}$; $d\mathbf{l}/dt = \boldsymbol{\tau}$ | §6.7.2, Eqs. 6.25a, 6.27, pp. 106–107 |
| Newton's 2nd law (rotation) | $\boldsymbol{\tau}_{\text{ext}} = d\mathbf{L}/dt$ | §6.7, Eq. 6.28b, p. 108 |
| Conservation of \mathbf{L} | $\boldsymbol{\tau}_{\text{ext}} = \mathbf{0} \Rightarrow \mathbf{L} = \text{constant}$ | §6.7, Eq. 6.29a, p. 108 |
| Moment of inertia | $I = \Sigma m_i r_i^2$ | §6.9, Eq. 6.34, p. 114 |
| Rotational KE | $K = \frac{1}{2} I \omega^2$ | §6.9, Eq. 6.35, p. 114 |

| Quantity | Symbol / Formula | NCERT reference |
|--|--|--------------------------------|
| Radius of gyration | $I = M k^2$ | §6.9, p. 115 |
| Newton's 2nd law (fixed axis) | $\tau = I \alpha$ | §6.11, Eq. 6.41, p. 120 |
| Work in rotation | $dW = \tau d\theta; P = \tau \omega$ | §6.11, Eqs. 6.39, 6.40, p. 119 |
| Angular momentum (fixed axis) | $L_z = I \omega$ | §6.12, Eq. 6.42b, p. 121 |
| Conservation about fixed axis | $I \omega = \text{constant}$ | §6.12.1, Eq. 6.44, p. 122 |
| Kinematic equations (constant α) | $\omega = \omega_0 + \alpha t; \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ | §6.10, Eqs. 6.36–6.37, p. 117 |
| Standard I (thin rod, \perp axis at mid) | $ML^2/12$ | Table 6.1, p. 116 |
| Standard I (solid sphere, diameter) | $(2/5)MR^2$ | Table 6.1, p. 116 |

Practice MCQs

PYQ Alignment

This chapter is one of the most reliably tested in CUET Physics — typically 10–12 MCQs per paper across CUET 2023–2025, spread across direct moment-of-inertia recall from Table 6.1, the $\tau = I\alpha$ and $K = \frac{1}{2}I\omega^2$ numericals (mirroring Examples 6.11–6.12), conceptual items on CM motion under internal forces, and conservation-of-angular-momentum scenarios (skater / swivel chair). Assertion–reason items on the equilibrium of a couple and on the τ –work dimensional coincidence recur regularly.

CUET 2023–25 — Actual PYQs from this chapter

No PYQs from this chapter appeared in CUET 2023, 2024 or 2025.