

CUET · PHYSICS · CLASS XII · CODE 322

# Atoms

CUET unit: Atoms

By UniDrill · NCERT-grounded study material

[WWW.UNIDRILL.IN](http://WWW.UNIDRILL.IN)

UniDrill

## Snapshot

- Traces the evolution of atomic models — Thomson's plum-pudding, Rutherford's nuclear (planetary) model, and Bohr's quantised-orbit model — using the Geiger-Marsden  $\alpha$ -scattering experiment as the pivotal evidence.
- Establishes that the atom is mostly empty space, with all positive charge and nearly all mass packed into a nucleus of size  $\sim 10^{-15}$ – $10^{-14}$  m, roughly  $10^4$ – $10^5$  times smaller than the atom ( $\sim 10^{-10}$  m).
- Resolves the instability problem of Rutherford's model through Bohr's three postulates: stable stationary orbits, angular-momentum quantisation  $L = nh/2\pi$ , and photon emission/absorption via  $h\nu = E_i - E_f$ .
- Yields the hallmark hydrogen-atom results —  $E_n = -13.6/n^2$  eV, ground-state Bohr radius  $a_0 = 0.53$  Å, ionisation energy 13.6 eV — and explains the angular-momentum quantisation through de Broglie standing waves ( $2\pi r_n = n\lambda$ ).
- Anchors the CUET unit "Atoms": NTA repeatedly tests scattering geometry, energy-level arithmetic, postulate identification, and the limitations of the Bohr model.

## Detailed Notes

### 2.1 Core concepts

Atomic theory at the close of the nineteenth century stood as follows. Once J. J. Thomson had discovered the electron in 1897, the question became: how are the electrons arranged inside a neutral atom? Thomson proposed the **plum-pudding model** in 1898 (NCERT §12.1, p. 290): a uniform sphere of positive charge throughout the volume of the atom, with electrons embedded in it like seeds in a watermelon, just dense enough to make the atom neutral overall. The model could roughly account for the size of atoms ( $\sim 1$  Å) and was consistent with the existence of electrons, but it had no real explanation for the most striking experimental fact about atoms — their **line spectra**. An excited rarefied atomic gas does not emit a continuous rainbow of colours but a sharp set of **discrete wavelengths** characteristic of the element. Solid or condensed matter, by contrast, gives a continuous spectrum (NCERT §12.1, p. 290–291). The spectrum of an element is its fingerprint, and any successful atomic model has to predict it.

The decisive experimental input came from the **Geiger-Marsden  $\alpha$ -scattering experiment** designed by Rutherford and carried out by his students Hans Geiger and Ernest Marsden between 1908 and 1911 (NCERT §12.2, p. 291–292). The setup used 5.5 MeV  $\alpha$ -particles emitted by a radioactive source of  $^{214}_{83}\text{Bi}$ . A pair of lead bricks collimated the  $\alpha$ -beam onto a very thin ( $\sim 2.1 \times 10^{-7}$  m) gold foil suspended in vacuum. Scattered  $\alpha$ -particles produced flashes of light (scintillations) on a ZnS screen viewed through a microscope mounted on an arm that could rotate around the foil to cover all scattering angles  $\theta$  from  $0^\circ$  to  $180^\circ$ .

The numbers Geiger and Marsden recorded surprised everyone. Most  $\alpha$ -particles (>99.86%) passed straight through with little or no deflection; only  $\sim 0.14\%$  scattered through angles greater than  $1^\circ$ ; and remarkably, **about 1 in 8000 was scattered through more than  $90^\circ$** , with a few even bouncing straight back along the direction of incidence. Rutherford famously remarked that the result was "as if you fired a fifteen-inch shell at a piece of tissue paper and it came back and hit you." Thomson's plum-pudding picture could not account for these violent back-scatterings — a diffuse cloud of positive charge would deflect  $\alpha$ -particles only very gradually.

Rutherford's interpretation was decisive (NCERT §12.2, p. 293). The  $\alpha$ -particle is a helium nucleus with charge  $+2e$  and mass  $\sim 7300$  times that of an electron. The electrons in the atom can scarcely deflect such a massive projectile. Large-angle scattering must come from a single close encounter with a much heavier object — a concentrated positive **nucleus** containing nearly all the atomic mass. The frequency of large-angle events told Rutherford that this nucleus was small: of order  $10^{-15}$  to  $10^{-14}$  m across, roughly  $10^4$ – $10^5$  times **smaller** than the atom ( $\sim 10^{-10}$  m). Hence the atom is mostly **empty space**, with electrons orbiting at distances vastly larger than the nuclear size — the **nuclear (planetary) model**.

The geometry of  $\alpha$ -scattering depends on the **impact parameter  $b$**  — the perpendicular distance of the  $\alpha$ -particle's incoming velocity vector from the centre of the target nucleus (NCERT §12.2.1, p. 293–294). Small  $b \Rightarrow$  very close encounter  $\Rightarrow$  large scattering angle  $\theta$ . The limiting case  $b = 0$  (head-on collision) gives back-scattering  $\theta \approx \pi$ , where the  $\alpha$ -particle decelerates to a momentary halt at the **distance of closest approach**  $d$  before being reversed by the Coulomb repulsion. Setting kinetic energy equal to electrostatic potential energy at the closest point,  $K = (1/4\pi\epsilon_0)(2e)(Ze)/d$ , gives  **$d = 2Ze^2/(4\pi\epsilon_0 K)$** . For a 7.7 MeV  $\alpha$ -particle striking a gold nucleus ( $Z = 79$ ),  $d \approx 3 \times 10^{-14}$  m = 30 fm, providing an upper bound on the gold-nuclear radius (NCERT Example 12.2, p. 295).

**Electron orbits in the Rutherford atom (NCERT § 12.2.2, p. 295–296).** For a circular orbit of an electron around a nucleus of charge  $+Ze$ , the Coulomb attraction supplies the centripetal force:

$$(1/4\pi\epsilon_0) Ze^2 / r^2 = m v^2 / r \Rightarrow v^2 = Ze^2/(4\pi\epsilon_0 m r).$$

$$KE = (1/2) m v^2 = Ze^2/(8\pi\epsilon_0 r), PE = -Ze^2/(4\pi\epsilon_0 r), \text{ and total energy}$$

$$E = KE + PE = -Ze^2/(8\pi\epsilon_0 r) \text{ (NCERT Eq. 12.4, p. 296).}$$

The total energy is **negative**, reflecting that the electron is bound; it would need to be supplied with  $|E|$  of energy to escape the atom. For hydrogen ( $Z = 1$ ) and the smallest stable orbit one would calculate,  $E$  comes out to about  $-13.6$  eV — quite close to the experimentally measured ionisation energy of hydrogen.

But Rutherford's atom has a fatal flaw (NCERT §12.4, p. 297). According to classical electromagnetic theory, an **accelerating** charge — and an electron in circular orbit is centripetally accelerating — must radiate electromagnetic waves continuously, losing energy. The electron would therefore spiral inward in about  $10^{-10}$  s, and the atom would **collapse** into the nucleus. Worse, the continuously changing orbital frequency would produce a continuous spectrum, contradicting the discrete line spectra observed for atoms.

Neither of these contradictions could be resolved within classical physics. Niels Bohr in 1913 proposed three radical postulates (NCERT §12.4, p. 298–299) that broke from classical EM in just the right way to save the planetary model.

**Postulate 1 — Stationary states.** An electron in an atom occupies certain stable orbits in which it does **not** radiate, contrary to classical EM. Each such orbit has a definite total energy. The classical instability problem is simply postulated away in these specific orbits.

**Postulate 2 — Angular-momentum quantisation.** The stationary orbits are those for which the orbital angular momentum is an integer multiple of  $\hbar = h/(2\pi)$ :

$$L = m v r = n h/(2\pi) = n\hbar, n = 1, 2, 3, \dots$$

where  $n$  is the **principal quantum number**.

**Postulate 3 — Frequency condition for transitions.** Photons are emitted or absorbed when the electron jumps between two stationary states of energies  $E_i$  and  $E_f$ , with the photon frequency given by

$$h \nu = E_i - E_f$$

(emission if  $E_i > E_f$ , absorption if  $E_i < E_f$ ).

These three postulates, combined with the Rutherford-orbit dynamics, completely specify the hydrogen atom. Solving Coulomb's law for circular orbit,  $mv^2/r = e^2/(4\pi \epsilon_0 r^2)$ , together with quantisation  $mvr = n\hbar$ , gives the **quantised radii**

$$r_n = (n^2/m)(h/2\pi)^2 (4\pi \epsilon_0/e^2) \text{ (NCERT Eq. 12.7, p. 299).}$$

For  $n = 1$  this evaluates to the famous **Bohr radius**

$$a_0 = 5.29 \times 10^{-11} \text{ m} \approx 0.53 \text{ \AA} \text{ (NCERT p. 300).}$$

Higher orbits scale as  $r_n = n^2 a_0$ :  $r_2 = 4 a_0$ ,  $r_3 = 9 a_0$ , and so on — the orbits get rapidly larger as  $n$  increases.

The corresponding **quantised energies** are

$$E_n = - m e^4 / (8 n^2 \epsilon_0^2 h^2) = -13.6/n^2 \text{ eV} \text{ (NCERT Eq. 12.10, p. 299).}$$

So  $E_1 = -13.6$  eV (ground state),  $E_2 = -3.40$  eV,  $E_3 = -1.51$  eV,  $E_4 = -0.85$  eV,  $\dots E_\infty = 0$  (NCERT Fig. 12.7, p. 300). Several immediate consequences follow:

- The **ionisation energy** of hydrogen from its ground state is  $|E_1| = 13.6$  eV — in excellent agreement with measurement.
- The **excitation energy** from  $n = 1$  to  $n = 2$  is  $E_2 - E_1 = 10.2$  eV; from  $n = 1$  to  $n = 3$  it is 12.09 eV.
- The energy levels get **closer together** as  $n$  grows (the spacing  $E_{\{n+1\}} - E_n$  shrinks).
- The orbital speed  $v_n \propto 1/n$ ;  $v_1 \approx c/137$  (where  $1/137$  is the fine-structure constant), so electrons in higher orbits move more slowly.

**Line spectra explained (NCERT § 12.5, p. 300–301).** When the atom is excited (by heat, light, or collisions) into a state  $n_i$  and then drops to a lower state  $n_f$ , the emitted photon has  $h \nu = E_{\{n_i\}} - E_{\{n_f\}}$ . Because the energies are discrete, so are the frequencies — only certain definite wavelengths appear. Conversely, when white light is sent through a cold gas, the same set of wavelengths is **absorbed**, producing dark **absorption lines** at exactly the same wavelengths the gas would emit. Bohr's frequency condition unifies emission and absorption in a single energy-conservation statement.

**de Broglie's interpretation (NCERT § 12.6, p. 301–302).** Why should angular momentum be quantised in units of  $h/(2\pi)$ ? Louis de Broglie in 1923 offered a striking explanation. Every moving particle has an associated matter wave of wavelength  $\lambda = h/(mv)$ . For a circular orbit to be stable, the wave must close on itself — the orbit must contain an **integer number of de Broglie wavelengths**:

$$2\pi r_n = n \lambda = n h/(m v_n).$$

Rearranging gives  $m v_n r_n = n h/(2\pi)$  — Bohr's quantisation condition! Bohr's stationary orbits are simply standing matter waves on circular orbits (NCERT Fig. 12.8, p. 301, shows the  $n = 4$  standing wave).

**Limitations of Bohr's model (NCERT § 12.6, p. 302).** Spectacularly successful as it is for hydrogen, Bohr's model fails the moment we add a second electron. It works only for **hydrogenic atoms** — single-electron systems like H,  $\text{He}^+$ ,  $\text{Li}^{2+}$  — because it ignores electron–electron Coulomb repulsion. It also fails to predict the **relative intensities** of spectral lines, a quantitative weakness. These shortcomings would eventually motivate the full quantum-mechanical treatment by Schrödinger, Heisenberg and others in 1925–26.

## 2.2 Definitions to memorise

Term	Definition	Page
Plum-pudding model		290

Term	Definition	Page
	Thomson's 1898 model: positive charge spread uniformly through the atom with electrons embedded like seeds in a watermelon	
Nuclear (planetary) model	Rutherford's model: all positive charge and almost all mass in a small central nucleus; electrons revolve around it	293
$\alpha$ -particle	Helium nucleus (2 protons + 2 neutrons), charge $+2e$ , mass $\approx 4 u$	291
Impact parameter (b)	Perpendicular distance of the initial velocity vector of the $\alpha$ -particle from the centre of the nucleus	294
Distance of closest approach (d)	Centre-to-centre distance $d$ between an $\alpha$ -particle and a nucleus at which the $\alpha$ -particle momentarily stops; $d = \frac{2Ze^2}{4\pi \epsilon_0 K}$	295
Scattering angle ( $\theta$ )	Angle between the incoming and outgoing directions of an $\alpha$ -particle in scattering	294
Emission line spectrum	Spectrum of bright discrete lines on a dark background emitted by a rarefied atomic gas excited at low pressure	296
Absorption spectrum	Dark lines in a continuous spectrum at exactly those wavelengths that the gas would emit	297
Stationary state	A stable orbit in Bohr's atom in which the electron revolves without emitting radiant energy	298
Principal quantum number (n)	Positive integer labelling Bohr orbits in ascending order of energy; appears in $L = nh/2\pi$	299
Angular-momentum quantisation	$L = m v r = n h/(2\pi)$	299
Frequency condition	$h \nu = E_i - E_f$ for photon emission/absorption	299
Ground state	Lowest-energy stationary state ( $n = 1$ ); for hydrogen, energy = $-13.6 \text{ eV}$	300
Excited state	Any stationary state with $n > 1$	300
Ionisation energy	Minimum energy required to free the electron from the ground state of the atom (13.6 eV for hydrogen)	300
Excitation energy	Energy difference between an excited state and the ground state	300
Bohr radius ( $a_0$ )	Radius of the $n = 1$ orbit of hydrogen; $a_0 \approx 5.3 \times 10^{-11} \text{ m} = 0.53 \text{ \AA}$	300
Bohr orbit radius	$r_n = n^2 a_0$ for hydrogen	299
Bohr energy	$E_n = -13.6/n^2 \text{ eV}$ for hydrogen	299

Term	Definition	Page
Hydrogenic atom	An atom or ion with a nucleus of charge $+Ze$ and a single electron — H, $\text{He}^+$ , $\text{Li}^{2+}$ , ...	302
de Broglie standing wave	$2\pi r_n = n\lambda$ , where $\lambda = h/(m v_n)$	301
Bohr fine-structure constant	$\alpha = e^2/(4\pi \epsilon_0 \hbar c) \approx 1/137$	implied via $v_1 \approx c/137$
Reduced Planck constant	$\hbar = h/(2\pi) \approx 1.05 \times 10^{-34} \text{ J s}$	299
Rydberg energy	13.6 eV, the ground-state binding energy of hydrogen	300

### 2.3 Diagrams / processes to remember

- **Fig. 12.1 (p. 292):** Geiger–Marsden scattering apparatus —  $\alpha$ -source, collimating lead bricks, thin gold foil, rotatable ZnS screen and microscope, all in a vacuum chamber.
- **Fig. 12.2 (p. 292):** Schematic arrangement of the experiment, showing the lead-brick collimator and the angular distribution of scattered  $\alpha$ -particles.
- **Fig. 12.3 (p. 293):** Plot of  $N(\theta)$  — number of  $\alpha$ -particles scattered per unit angle — vs scattering angle  $\theta$ . Experimental points (dots) match Rutherford's nuclear-model curve over many orders of magnitude.
- **Fig. 12.4 (p. 294):**  $\alpha$ -particle trajectories in the Coulomb field of the nucleus for several different impact parameters  $b$ , showing how  $\theta$  grows as  $b$  shrinks.
- **Fig. 12.5 (p. 297):** Emission lines in the visible (Balmer) series of hydrogen.
- **Fig. 12.6 (p. 298):** Spiral inward path predicted by classical electromagnetism for an accelerating orbiting electron — the instability of Rutherford's model.
- **Fig. 12.7 (p. 300):** Energy-level diagram for hydrogen: horizontal lines at  $-13.6 \text{ eV}$  ( $n = 1$ ),  $-3.4 \text{ eV}$  ( $n = 2$ ),  $-1.51 \text{ eV}$  ( $n = 3$ ), ... converging to  $0 \text{ eV}$  at  $n = \infty$ , with vertical arrows showing emission transitions.
- **Fig. 12.8 (p. 301):** Standing de Broglie wave on a circular orbit with  $n = 4$  ( $2\pi r_n = 4\lambda$ ).

### 2.4 Common confusions / NTA trap points

- Confusing the two failures of Rutherford's model — instability of the atom (electron spirals in) vs failure to explain line spectra. Both arise from classical EM; NTA likes to test which postulate of Bohr's plugs which hole (p. 297).
- Mixing up the  $n$ -dependence:  $r_n \propto n^2$  (radius grows),  $v_n \propto 1/n$ ,  $E_n \propto -1/n^2$  (becomes less negative as  $n$  grows). As  $n$  increases, energy levels get closer together.

- Sign of total energy:  $E_n$  is **negative** because the electron is bound; "energy required to ionise" is the **magnitude** (13.6 eV for ground state). Do not confuse the kinetic energy (positive) with total energy (negative).
- The energy **difference** between ground ( $n = 1$ ) and first excited ( $n = 2$ ) state is **10.2 eV**, NOT 3.4 eV (which is  $E_2$  alone). Similarly  $E_3 - E_1 = 12.09$  eV.
- Bohr's model is valid only for **hydrogenic** (one-electron) atoms — H,  $\text{He}^+$ ,  $\text{Li}^{2+}$ , etc. It breaks for any atom with two or more electrons.
- The  $\alpha$ -particle in Geiger–Marsden does NOT physically touch the nucleus; the distance of closest approach (e.g. 30 fm for 7.7 MeV  $\alpha$  on gold) is larger than the sum of the radii.
- The fraction "1 in 8000" refers specifically to deflections greater than  $90^\circ$ . "0.14%" is the fraction deflected greater than  $1^\circ$ .
- Bohr's angular-momentum quantum is  $h/(2\pi)$  — not  $h$ , and not  $h/\pi$ .
- The frequency condition uses  $|E_i - E_f|$ ; if  $E_i < E_f$  the atom **absorbs** a photon, not emits.
- Distance of closest approach  $d$  depends on  $K$  (the kinetic energy of the  $\alpha$ -particle) — doubling the kinetic energy halves  $d$ .
- For an emission line, the wavelength is the same whether you label the transition as  $n_i \rightarrow n_f$  or  $n_f \rightarrow n_i$ ; absorption produces the same line.
- Bohr orbits are **not** electron clouds; they are circular orbits in a semi-classical picture. The full quantum-mechanical picture replaces them with probability densities.

## 2.5 Key formulas table

Symbol	Formula	Meaning	NCERT page
Coulomb scattering	$F = (1/4\pi\epsilon_0)(2Ze^2/r^2)$	$\alpha$ -nucleus repulsion	293
Closest approach	$d = 2Ze^2/(4\pi\epsilon_0 K)$	Centre-to-centre minimum distance	295
Orbit force balance	$mv^2/r = e^2/(4\pi\epsilon_0 r^2)$	Centripetal = Coulomb	295
KE (Rutherford)	$KE = e^2/(8\pi\epsilon_0 r)$	Half the magnitude of PE	296
PE (Rutherford)	$PE = -e^2/(4\pi\epsilon_0 r)$	Negative, bound	296
Total E (Rutherford)	$E = -e^2/(8\pi\epsilon_0 r)$	Total = $-KE = PE/2$	296, Eq. 12.4
Bohr postulate 1	Stable stationary orbits	No radiation in these orbits	298
Bohr postulate 2	$L = mvr = nh/(2\pi)$		

Symbol	Formula	Meaning	NCERT page
		Angular momentum quantised	299, Eq. 12.5
Bohr postulate 3	$h\nu = E_i - E_f$	Frequency condition	299, Eq. 12.6
Orbit radius	$r_n = (n^2 \epsilon_0 h^2) / (\pi m e^2)$	Quantised radius (hydrogen)	299, Eq. 12.7
Bohr radius	$a_0 = 0.53 \text{ \AA}$	$n = 1$ hydrogen radius	300
$r_n$ hydrogen	$r_n = n^2 a_0$	Higher orbits	300
Orbital speed	$v_n = e^2 / (2 \epsilon_0 n h)$	Decreases with $n$	299
Energy (hydrogen)	$E_n = -me^4 / (8 \epsilon_0^2 n^2 h^2)$	Bohr energy levels	299, Eq. 12.10
Energy numeric	$E_n = -13.6/n^2 \text{ eV}$	Rydberg form	300
Ground-state energy	$E_1 = -13.6 \text{ eV}$	$n = 1$	300
Ionisation energy	13.6 eV (hydrogen)	From ground state	300
Excitation 1→2	$E_2 - E_1 = 10.2 \text{ eV}$	First excitation	300
Excitation 1→3	$E_3 - E_1 = 12.09 \text{ eV}$	Second excitation	300
de Broglie standing wave	$2\pi r_n = n \lambda$	Wave-fit condition	301, Eq. 12.12
de Broglie wavelength	$\lambda = h / (m v_n)$	Matter wave	301
Hydrogenic generalisation	$E_n = -13.6 Z^2/n^2 \text{ eV}$	For H, He <sup>+</sup> , Li <sup>2+</sup> etc.	302 (implied)
Hydrogenic radius	$r_n = (n^2/Z) a_0$	Z-scaled Bohr radius	302

## Practice MCQs

## PYQ Alignment

Atoms is a high-yield CUET unit — across CUET (UG) 2023–25 papers it consistently delivers ~10 MCQs per year. NTA's favourite question types from this chapter are: (i) numerical recall of Bohr quantities (radii, energies, transitions, ionisation energy of hydrogen); (ii) statement/assertion-reason items on Bohr's three postulates and the limitations of the Bohr and Rutherford models; (iii) conceptual items on the Geiger–Marsden experiment — observed fractions, conclusions, role of the impact parameter,

distance of closest approach; and (iv) match-the-following on n-dependence of  $r_n$ ,  $v_n$ , and  $E_n$ .

### CUET 2025 — Actual PYQs from this chapter

**Q.44 (CUET 2025)** An electron in the ground state of hydrogen absorbs 12.09 eV. The angular momentum increases by:

- A)  $h/2\pi$  B)  $2h/2\pi$  C)  $3h/2\pi$  D)  $4h/2\pi$  Tests: Hydrogen atom — transition from  $n=1$  absorbing 12.09 eV (to  $n=3$ ),  $\Delta L = 2(h/2\pi)$  Answer: Not in extracted key

### CUET 2024 — Actual PYQs from this chapter

**Q.26 (CUET 2024)** Electron KE in ground state = K. Potential and total energy:

- A)  $-2K$ ,  $-K$  B)  $+2K$ ,  $-K$  C)  $-K$ ,  $+2K$  D)  $+K$ ,  $+2K$  Tests: Hydrogen atom — KE, PE, total energy relations ( $E = -K$ ) Answer: Not in extracted key

**Q.28 (CUET 2024)** Shortest wavelengths in hydrogen series decreasing order:

- A) Pfund, Balmer, Brackett, Lyman B) Pfund, Brackett, Balmer, Lyman C) Balmer, Pfund, Lyman, Brackett D) Pfund, Brackett, Lyman, Balmer Tests: Hydrogen spectral series — short-wavelength limits Answer: Not in extracted key

**Q.48 (CUET 2024)** According to Bohr's model, which are correct? (A) Radius  $\propto n^2$  (B) Speed  $\propto 1/n$  (C) Energy  $\propto -1/n^2$  (D) Radius  $\propto n$  Options: 1. A, B, C 2. A, B, D 3. A, B, C, D 4. B, C, D

- (options not in extracted source — see official paper) Tests: Bohr model — radius  $\propto n^2$ , speed  $\propto 1/n$ , energy  $\propto -1/n^2$  Answer: Not in extracted key

### CUET 2023 — Actual PYQs from this chapter

**Q.38 (CUET 2023)** Energy levels of hydrogen atom are shown. Which wavelength corresponds to the transition shown? (Options correspond to the labelled transitions in the diagram.)

- (options not in extracted source — see official paper) Tests: Hydrogen energy-level transitions — wavelengths Answer: Not in extracted key

**Q.50 (CUET 2023)** Match List-I with List-II using Bohr's atomic model. List I List II Radius of electron orbit Directly proportional to ( $n^2$ ) Angular momentum ( $nh/2\pi$ ) Velocity of electron Inversely proportional to ( $n$ ) Energy of electron Inversely proportional to ( $n^2$ ) Options:

- A) A-I, B-II, C-III, D-IV B) A-II, B-I, C-III, D-IV C) A-IV, B-III, C-II, D-I D) A-I, B-III, C-II, D-IV Tests: Bohr model — radius, angular momentum, velocity, energy with  $n$  Answer: Not in extracted key