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CUET · PHYSICS · CLASS XII · CODE 322

Electrostatic Potential and Capacitance

CUET unit: Electrostatic Potential and Capacitance

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📌 Snapshot

- Defines electrostatic potential V as work done per unit positive test charge in moving it (without acceleration) from infinity to the point, with potential at infinity chosen to be zero.
- Derives $V = Q/(4\pi\varepsilon_0 r)$ for a point charge, $V = (1/4\pi\varepsilon_0)(p\cdot\hat{r})/r^2$ for a short dipole, and uses superposition for a system of charges.
- Establishes the field–potential relation $E = -dV/dl$ (field points along steepest decrease of V) and the geometry of equipotential surfaces.
- Lays out the five-result electrostatics of conductors ($E = 0$ inside; surface $E = \sigma/\varepsilon_0\,\hat{n}$; constant V throughout; charge resides on outer surface; shielded cavity) and introduces dielectrics with $P = \varepsilon_0\chi_e E$.
- Builds capacitance $C = Q/V$, derives $C = \varepsilon_0 A/d$ for a parallel-plate capacitor, shows $C = K\varepsilon_0 A/d$ with a dielectric, gives series/parallel combinations, and stores energy $U = \frac{1}{2}CV^2 = Q^2/(2C) = \frac{1}{2}QV$ with energy density $u = \frac{1}{2}\varepsilon_0 E^2$.

📖 Detailed Notes

2.1 Core concepts

The Coulomb force between two point charges, like the gravitational force between two masses, is conservative: its inverse-square form guarantees that the work done in moving a charge between two fixed end-points is independent of the path taken. This single mathematical fact lets us define a potential energy function U(r) for a charge in an electrostatic field, in exact analogy with the gravitational potential energy mgh near Earth's surface (NCERT §2.1, p. 45–46). The electrostatic potential energy difference between two points P and R is defined as the work done by an external agency, slowly and without producing any acceleration, in moving a test charge q from R to P:

$\Delta U = U_P - U_R = W_{RP}$ (NCERT Eq. 2.2, p. 46).

Dividing through by the test charge q (and taking q to be a unit positive charge as a limiting procedure) gives the **electrostatic potential V** — work per unit positive test charge brought from infinity to the point in question, with V at infinity chosen to be zero (NCERT §2.2, p. 47, Eq. 2.4). Only potential differences are physically meaningful; the absolute V depends on the (conventional) choice of zero.

For a point charge Q at the origin, integrating the Coulomb force along any path from infinity to a point at distance r gives

$V(r) = Q/(4\pi\varepsilon_0 r)$ (NCERT Eq. 2.8, p. 48).

The potential is positive for positive Q and negative for negative Q; it falls off as 1/r, while the electric field falls off as $1/r^2$ (NCERT Fig. 2.4, p. 49). Note carefully: V is a scalar, so for a system of charges it is added algebraically with the signs of the charges included, not as a vector — a crucial simplification compared with the field calculation.

Dipole potential (NCERT §2.4, p. 49–50). An electric dipole consists of two equal and opposite charges +q and −q separated by a small distance 2a, with dipole moment p = 2qa directed from −q to +q. For a point at distance r from the centre with r » a, geometry yields

$V = (1/4\pi\varepsilon_0)\,(p\cdot\hat{r})/r^2 = p\cos\theta/(4\pi\varepsilon_0 r^2)$ (NCERT Eq. 2.15, p. 50),

where θ is the angle between p and the position vector. Two observations stand out. First, V falls off as $1/r^2$ for a dipole — faster than the 1/r decay of a point charge — because of the partial cancellation between the +q and −q contributions. Second, on the **axial line** ($\theta = 0$ or π) $V = \pm p/(4\pi\varepsilon_0 r^2)$ (maximum magnitude), while on the **equatorial plane** ($\theta = \pi/2$) **V = 0** even though E is non-zero. This is a common conceptual trap: V = 0 does not imply E = 0.

For a system of n point charges, superposition gives $V = (1/4\pi\varepsilon_0)\ \Sigma\ q_i/r_{iP}$. For a uniformly charged thin spherical shell of total charge q and radius R, integration (or Gauss's law combined with $V = -\int E\cdot dl$) yields $V = q/(4\pi\varepsilon_0 r)$ outside and **$V = q/(4\pi\varepsilon_0 R)$ inside (constant)** — the same as the surface value, even though the field inside is zero (NCERT §2.5, p. 51–52, Eq. 2.19).

Equipotential surfaces (NCERT §2.6, p. 54). An equipotential surface is a surface on which V has the same value at every point. Several consequences follow immediately. (i) The electric field E must be perpendicular (normal) to the equipotential surface at every point, because any tangential component would do work moving a charge along the surface — contradicting $\Delta V = 0$. (ii) No work is done in moving a test charge along an equipotential. (iii) Two equipotentials cannot intersect (else E would have two directions at the intersection). For a single point charge equipotentials are concentric spheres; for a uniform field they are parallel planes perpendicular to the field; for a dipole they look like distorted figure-eights; for two equal positive charges they form a characteristic dumbbell pattern.

Field–potential relation (NCERT §2.6.1, p. 55). If two closely spaced equipotentials have potentials V and $V + \delta V$ separated by perpendicular distance δl, the work done moving a unit test charge perpendicular to the surface is $qE\,\delta l = -\delta V$; cancelling and taking the limit,

$E = -dV/dl$ (NCERT Eq. 2.21, p. 55).

The minus sign says the field points in the direction of steepest decrease of V. Equivalently, in vector form $E = -\nabla V$ — the field is the negative gradient of the potential.

Potential energy of a system of charges (NCERT §2.7, p. 55–56). Bringing two charges q_1, q_2 from infinity to separation r_{12} requires external work

$U_{12} = (1/4\pi\varepsilon_0)(q_1q_2/r_{12})$ (Eq. 2.22).

For three charges, $U = (1/4\pi\varepsilon_0)(q_1q_3/r_{12} + q_1q_3/r_{13} + q_2q_3/r_{23})$ — one term per distinct pair (Eq. 2.26). U is positive for like charges (work has to be done against repulsion) and negative for unlike (system gains energy when assembled).

For a charge q sitting in an external field that produces potential V(r), the potential energy is U = qV(r) (Eq. 2.27, p. 58). One electron volt (eV) is defined as the energy gained by an electron crossing a potential difference of 1 V: **1 eV = 1.6 × 10^{-19} J** (NCERT §2.8.1, p. 59).

Dipole in external field (NCERT §2.8.3, p. 60). A dipole of moment p in a uniform external E experiences zero net force but a torque $\tau = p \times E$. The work done rotating the dipole from $\theta_0 = \pi/2$ to θ is

$U(\theta) = -pE\cos\theta = -p\cdot E$ (NCERT Eq. 2.32, p. 60),

minimum (= −pE, stable) at $\theta = 0$ (p parallel to E) and maximum (= +pE, unstable) at $\theta = \pi$ (anti-parallel).

Electrostatics of conductors (NCERT §2.9, p. 61–63). Five key results follow from the fact that the free electrons in a conductor in equilibrium have zero net drift:

1. **E = 0 inside a conductor** in electrostatic equilibrium.
2. The **electric field at the surface is normal** to the surface — any tangential component would drive a surface current.
3. **All excess charge resides on the outer surface** of a conductor (Gauss's law applied to an interior Gaussian surface gives zero enclosed charge).
4. **V is constant** throughout the body and surface of a conductor — the entire conductor is one equipotential.
5. The **surface field** is $E = \sigma/\varepsilon_0\,\hat{n}$(Eq. 2.35, p. 63), derived using a Gaussian pill-box straddling the surface. Note the σ/ε_0 — not $\sigma/(2\varepsilon_0)$, which is the formula for an isolated infinite plane sheet.

A consequence is **electrostatic shielding**: the field inside a charge-free cavity within a conductor is zero, regardless of how the conductor is charged or what external fields are applied. This is why sensitive electronics are housed in metallic enclosures (Faraday cages).

Dielectrics (NCERT §2.10, p. 64–67). A dielectric is an insulator with no free charges, but its bound molecular dipoles can respond to an external field — non-polar molecules (CH_4) acquire an induced dipole moment, polar molecules (H_2O) align their

permanent dipoles. Either way, a polarised dielectric carries a dipole moment per unit volume P called the **polarisation**. For a linear, isotropic dielectric,

$P = \varepsilon_0 \chi_e E$ (NCERT Eq. 2.37, p. 66),

where χ_e is the dimensionless **electric susceptibility**. The induced surface charges on the dielectric reduce the net field inside it by a factor K, the **dielectric constant**, related to susceptibility by $K = 1 + \chi_e$.

Capacitance (NCERT §2.11, p. 67–68). A capacitor is any pair of conductors separated by an insulator. When charge Q is transferred from one conductor to the other (creating +Q on one, –Q on the other), a potential difference V appears between them, proportional to Q. The ratio

$C = Q/V$ (Eq. 2.38)

is the **capacitance**. SI unit is the **farad** ($F = C\ V^{-1}$); practical capacitors are rated in μF (10^{-6} F) or pF (10^{-12} F). C depends only on the geometry of the conductors and the dielectric between them — **not** on the charge stored or voltage applied. The maximum field a dielectric can support without breakdown — the **dielectric strength** — limits the charge a capacitor can hold; for air it is about $3 \times 10^6\ V\ m^{-1}$.

Parallel-plate capacitor (NCERT §2.12, p. 68–69). Two large plates of area A separated by $d \ll \sqrt{A}$ carry charges +Q and –Q. The field between them is $E = \sigma/\varepsilon_0 = Q/(\varepsilon_0 A)$; the potential difference is $V = Ed$; hence

$C = \varepsilon_0 A / d$ (Eq. 2.43, p. 69).

If the gap is fully filled with a dielectric of constant K, the field reduces to $\sigma/(K \varepsilon_0)$, V drops by factor K for the same Q, and

$C = K \varepsilon_0 A / d = K C_0$ (NCERT §2.13, Eq. 2.51, p. 70).

The general definition $K = C/C_0$ — ratio of capacitance with vs without the dielectric — applies to any capacitor geometry.

Combinations of capacitors (NCERT §2.14, p. 71–72). Two rules to remember:

- **Series**: same charge Q on each. The voltages add: $V = V_1 + V_2 + \cdots = Q(1/C_1 + 1/C_2 + \cdots)$. Equivalent **$1/C_s = \Sigma\ 1/C_i$** (Eq. 2.60).
- **Parallel**: same voltage V across each. The charges add: $Q = \Sigma\ Q_i = V\ \Sigma\ C_i$. Equivalent **$C_p = \Sigma\ C_i$** (Eq. 2.67).

Notice this is the **opposite** of the resistor rule — a classic exam trap.

Energy stored (NCERT §2.15, p. 73–75). Charging a capacitor amounts to transferring infinitesimal charge $\delta Q'$ at the instantaneous voltage Q'/C; integrating,

$U = Q^2/(2C) = \frac{1}{2} CV^2 = \frac{1}{2} Q V$ (Eq. 2.73, p. 74).

The energy is stored in the **field** between the plates, with **energy density $u = \frac{1}{2}\varepsilon_0 E^2$** (Eq. 2.73, p. 75) — a general result, valid in vacuum for any electrostatic configuration, not only parallel plates.

2.2 Definitions to memorise

Term	Definition	Page
Electrostatic potential V	Work done per unit positive test charge (without acceleration) in bringing it from infinity to that point	47
Potential difference ($V_P - V_R$)	Work done per unit positive charge by external force in moving it from R to P	47
Volt	SI unit of potential, $1\ V = 1\ J\ C^{-1}$	47
Equipotential surface	A surface with a constant value of potential at all points	54
Field-potential gradient relation	$E = -dV/dl$; $E = -\nabla V$	55
Electric dipole moment (p)	$p = 2qa$, vector from $-q$ to $+q$	50
Potential energy of two charges	$U = (1/4\pi\varepsilon_0)(q_1q_2/r_{12})$	56
Dipole PE in external field	$U = -p\cdot E$	60
Electron volt (eV)	Energy gained by an electron through a potential difference of 1 V; $1\ eV = 1.6 \times 10^{-19}\ J$	59
Conductor in equilibrium	Body in which $E_{internal} = 0$ and V is constant throughout	61
Surface charge density σ	Charge per unit area on a conductor's surface	63
Surface field on conductor	$E = (\sigma/\varepsilon_0)\,\hat{n}$	63
Electrostatic shielding	Vanishing of electric field inside a (charge-free) cavity of a conductor, irrespective of external fields	63
Dielectric	Insulator that polarises in an external field, no free charges	64
Polarisation P	Dipole moment per unit volume of a polarised dielectric	66
Electric susceptibility χ_e	Constant of proportionality in $P = \varepsilon_0 \chi_e E$ for a linear isotropic dielectric	66
Dielectric constant K	Ratio $\varepsilon/\varepsilon_0$; equivalently $K = C/C_0$	70
Dielectric strength	Maximum field a dielectric can withstand without breakdown ($\sim 3 \times 10^6\ V\ m^{-1}$ for air)	68
Capacitance C	Ratio Q/V; depends only on geometry and dielectric	67

Term	Definition	Page
Farad (F)	SI unit of capacitance; 1 F = 1 C V^{-1}	67
Series capacitors	$1/C_s = \Sigma\ 1/C_i$ (same charge on each)	72
Parallel capacitors	$C_p = \Sigma\ C_i$ (same voltage across each)	72
Energy stored	$U = Q^2/(2C) = ½CV^2$	74
Energy density u	$u = ½\ \varepsilon_0 E^2$	75

2.3 Diagrams / processes to remember

- **Fig. 2.3 / Fig. 2.4 (p. 48–49):** Variation of V (∝ 1/r) and E (∝ $1/r^2$) with distance from a point charge — useful for distinguishing the two graphs in distractor sets.
- **Fig. 2.5 (p. 50):** Geometry of potential due to a dipole (r_1, r_2, θ) — basis of $V = (p \cos\theta)/(4\pi\varepsilon_0 r^2)$.
- **Fig. 2.9–2.11 (p. 54):** Equipotential surfaces — concentric spheres for a point charge; parallel planes for a uniform field; characteristic patterns for dipole and two identical positive charges.
- **Fig. 2.12 (p. 55):** Two closely spaced equipotentials V and $V + \delta V$; derivation of $E = -\delta V/\delta l$.
- **Fig. 2.17 (p. 63):** Pill-box Gaussian surface used to prove $E = \sigma/\varepsilon_0\, \hat{n}$ at the surface of a charged conductor.
- **Fig. 2.19 (p. 64):** Summary diagram of the five electrostatic properties of a conductor.
- **Fig. 2.20–2.23 (p. 65–67):** Conductor vs dielectric in an external field; polar/non-polar molecules; bound surface charge $\pm\sigma_p$ reducing the net field.
- **Fig. 2.25 (p. 68):** Parallel-plate geometry — fields cancel outside, add to σ/ε_0 between the plates.
- **Fig. 2.26–2.28 (p. 71–72):** Series (same Q) and parallel (same V) combinations.
- **Fig. 2.30 (p. 74):** Capacitor charging as transfer of infinitesimal charge $\delta Q'$ at potential $V' = Q'/C$, integrated to $U = Q^2/(2C)$.

2.4 Common confusions / NTA trap points

- V varies as 1/r for a point charge but as $1/r^2$ for a short dipole — and E correspondingly as $1/r^2$ and $1/r^3$. Distractors swap these.
- Potential is a scalar; it is added algebraically (with signs of charges), not as vectors. The hexagon-of-charges and "V at centre of equilateral triangle of charges" type questions exploit this.
- On the equatorial plane of a dipole, V = 0 but E ≠ 0; on the axis E ≠ 0 and V ≠ 0. Students confuse "V = 0 ⇒ E = 0".

- Inside a charged conductor $E = 0$ but V is constant and non-zero — equals the surface value. Inside a charged spherical shell V is the same as on the surface, $V = q/(4\pi\varepsilon_0 R)$, not zero.
- When a dielectric is inserted with the battery **connected**, V is fixed and Q increases by factor K; when the battery is **disconnected**, Q is fixed and V decreases by factor K (so E inside the dielectric drops). NTA loves this distinction.
- Series combination: 1/C is additive (charge same on each); parallel: C is additive (V same across each). Students often invert this with the resistor rule.
- Capacitance depends only on geometry and the dielectric — not on the charge stored or the voltage applied.
- For a dipole in a uniform field, the net force is zero but the torque ($p \times E$) is not; $U = -p\cdot E$ is minimum when p is aligned with E (stable) and maximum when antiparallel (unstable).
- Surface field of a conductor is σ/ε_0 (not $\sigma/2\varepsilon_0$ — that is for an isolated infinite plane sheet).
- 1 eV = 1.6×10^{-19} J — a unit of energy, not voltage.
- For a uniform field over a distance l, $V = E \times l$ (not $E \times l^2$) — option (D) of Q6 below highlights this trap.
- Inserting a slab of thickness $t < d$ reduces the effective gap to $d - t + t/K$, a frequent CUET tweak.

2.5 Key formulas table

Symbol	Formula	Meaning	NCERT page
V (point charge)	$V = Q/(4\pi\varepsilon_0 r)$	Potential of point charge	48, Eq. 2.8
V (dipole)	$V = (1/4\pi\varepsilon_0)(p\cdot\hat{r})/r^2$	Far field of short dipole	50, Eq. 2.15
V (system)	$V = (1/4\pi\varepsilon_0)\ \Sigma$ q_i/r_iP	Superposition for n charges	51
V (shell, outside)	$V = q/(4\pi\varepsilon_0 r)$	Outside a charged shell	52
V (shell, inside)	$V = q/(4\pi\varepsilon_0 R)$ (constant)	Inside a charged shell	52, Eq. 2.19
E ↔ V	$E = -dV/dl$	Field is negative gradient of V	55, Eq. 2.21
U (two charges)	$U = (1/4\pi\varepsilon_0)(q_1q_2/r_{12})$	Pair PE	56, Eq. 2.22
U (three charges)	Σ over distinct pairs	Total PE	56, Eq. 2.26
U (q in ext. field)	$U = qV(r)$	Charge in external potential	58, Eq. 2.27
U (dipole)	$U = -p\cdot E$	Dipole in uniform external field	60, Eq. 2.32
τ (dipole)	$\tau = p \times E$	Torque on dipole	60

Symbol	Formula	Meaning	NCERT page
eV	$1\ \text{eV} = 1.6 \times 10^{-19}$ J	Energy unit	59
Conductor surface E	$E = (\sigma/\varepsilon_0)\ \hat{n}$	Just outside a conductor	63, Eq. 2.35
Polarisation	$P = \varepsilon_0 \chi_e E$	Linear dielectric	66, Eq. 2.37
Dielectric constant	$K = 1 + \chi_e = C/C_0$	Dimensionless	70
Capacitance	$C = Q/V$	Definition	67, Eq. 2.38
Parallel plate (vacuum)	$C_0 = \varepsilon_0 A/d$	Standard formula	69, Eq. 2.43
With dielectric	$C = K \varepsilon_0 A/d$	Dielectric fully filling gap	70, Eq. 2.51
Series	$1/C_s = \Sigma\ 1/C_i$	Same Q	72, Eq. 2.60
Parallel	$C_p = \Sigma\ C_i$	Same V	72, Eq. 2.67
Energy stored	$U = Q^2/(2C) = ½CV^2 = ½QV$	All equivalent	74, Eq. 2.73
Energy density	$u = ½ \varepsilon_0 E^2$	Energy per unit volume in E	75

🎯 Practice MCQs

📊 PYQ Alignment

This chapter is one of the highest-yield chapters in CUET Physics, typically contributing 10–13 MCQs per year across CUET 2023–2025. Question patterns repeat reliably: (i) direct formula recall — $V = kQ/r$, $U = ½CV^2$, $C = \varepsilon_0 A/d$, $E = \sigma/\varepsilon_0$; (ii) numerical plug-ins on series/parallel networks and energy stored (Examples 2.9, 2.10 and Exercises 2.5–2.11 are very common templates); (iii) conceptual statement-based items on equipotentials, the field–potential relation $E = -dV/dl$, and the five-result electrostatics of conductors; (iv) dielectric-insertion effects (battery connected vs disconnected) and the $C = KC_0$ identity; (v) dipole potential and $U = -p \cdot E$ in a uniform field.

CUET 2025 — Actual PYQs from this chapter

Q.2 (CUET 2025) A charge of magnitude 3×10^{-7} C is located at a distance of 0.09 m from a point P. Obtain the work done in bringing a charge 2×10^{-9} C from infinity to point P.

- A) 1.6×10^{-4} J B) 6×10^{-2} J C) 4×10^{-3} J D) 6×10^{-3} J Tests: Work done in bringing a charge from infinity to a point — $W = qV$ Answer: Not in extracted key

Q.3 (CUET 2025) In a series combination of capacitors connected across a battery:

- A) Each capacitor has equal charge for certain capacitance values only B) Each capacitor has different charge for certain capacitance values C) Each capacitor has equal charge for any value of capacitance D) Each capacitor has different charge for any value of capacitance Tests: Series capacitors — charge is the same on each Answer: Not in extracted key

Q.4 (CUET 2025) Two point charges +4 μC and −3 μC (with no external field) are located at (-6 cm, 0, 0) and (6 cm, 0, 0) respectively. The work required to separate them infinitely away from each other is:

- A) 0.9 J B) 0.18 J C) −0.9 J D) −0.018 J Tests: Work to separate two point charges to infinity = $-U_{initial}$ Answer: Not in extracted key

Q.6 (CUET 2025) A parallel plate capacitor having plates of area 200 cm^2 and separation 2.0 mm holds a charge 0.06 μC when 60 V potential difference is applied. The dielectric constant of the material between the plates is:

- A) 0.113 B) 1.13 C) 11.3 D) 113 Tests: Parallel-plate capacitor — dielectric constant from Q, V, A, d Answer: Not in extracted key

Q.7 (CUET 2025) The electric potential due to an electric dipole: (A) depends on r (B) depends on angle between position vector and dipole moment (C) falls off as $1/r^2$ (D) does not depend on separation between charges Choose the correct option:

- A) (A), (B), (C) B) (A), (B), (C), (D) C) (A), (B), (D) D) (B), (C), (D) Tests: Potential of an electric dipole — r and θ dependence, falls as $1/r^2$ Answer: Not in extracted key

CUET 2024 — Actual PYQs from this chapter

Q.2 (CUET 2024) Two capacitors 2 μF and 3 μF are in series across voltage V. Relation between potentials V_1, V_2 and energies U_1, U_2 is:

- A) $V_1/V_2 = U_1/U_2 = 2/3$ B) $V_1/V_2 = U_1/U_2 = 3/2$ C) $V_1/V_2 = 2/3$ and $U_1/U_2 = 3/2$ D) $V_1/V_2 = 3/2$ and $U_1/U_2 = 2/3$ Tests: Series capacitors — voltage ratio and energy ratio Answer: Not in extracted key

Q.34 (CUET 2024) A 4 mm thick dielectric slab is introduced between capacitor plates separated by 4 mm. To restore the original capacitance, plate separation must increase by 3.2 mm. The dielectric constant is:

- A) 2 B) 5 C) 3 D) 7 Tests: Dielectric slab in capacitor — effective separation and K Answer: Not in extracted key

CUET 2023 — Actual PYQs from this chapter

Q.4 (CUET 2023) An isolated sphere has a capacitance of 60 pF. What is the radius of the sphere?

- A) 540 cm B) 54 cm C) 0.054 cm D) 0.54 cm Tests: Capacitance of an isolated spherical conductor Answer: Not in extracted key

Q.6 (CUET 2023) Three capacitors of capacitances 2 μF, 6 μF and 12 μF are connected in series. If a 7 V battery is connected across the combination, find the potential difference across the 6 μF capacitor.

- A) 1 V B) 2 V C) 3 V D) 4 V Tests: Series combination of capacitors — charge and PD distribution Answer: Not in extracted key

Q.46 (CUET 2023) Two charges (+q) and (-3q) are kept 2 cm apart. Distance of the point where potential becomes zero:

- A) 6 cm B) 4 cm C) 3 cm D) 2 cm Tests: Zero-potential point for two unlike point charges Answer: Not in extracted key